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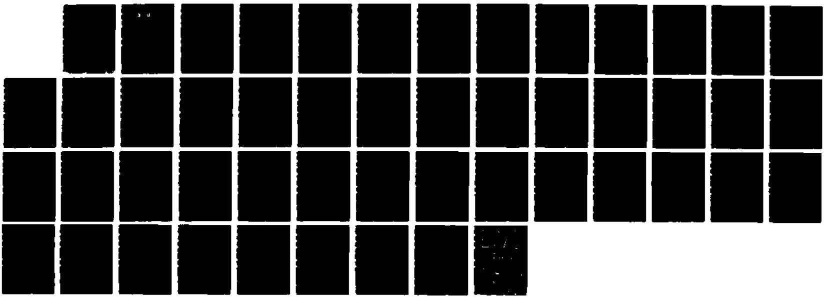

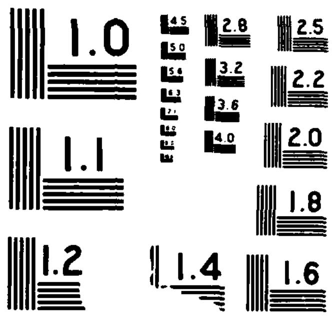
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PLASTIC YIELDING AT CRACK TIPS

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ABSTRACT

Small scale plastic yielding at crack tips is studied by means of nonlocal elasticity. Plastic lines along the crack line of Mode III crack are modelled by an array of dislocations. It is shown that plastic yield begins after a definite value of load, as a consequence of nonlocality. The length of plastic zone and the dislocation distrubution are determined as functions of the applied load. Results are in good agreement with experimental observations.

1. INTRODUCTION

One of the fundamental problems of fracture mechanics is the determination of elasto-plastic stress field near crack tips in elastic solids. Importance of this knowledge to designers needs no comments. Traditionally this problem is approached by means of classical plasticity theory which

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makes no reference to the internal structure of materials. However, it is well-known that the ultimate understanding of the crack initiation , plastic yielding and fracture process are intimately tied to the size and distibution of defects (such as impurities, voids, dislocations etc.) which exist naturally in materials. Consequently, the connection and interrelation of the crack-dislocation assemblies to the macroscopic plasticity and yielding are fundamental to the fracture mechanism. It is well-known that classical elasticty predicts an infinite stress at the tips of a line crack. This result prohibits the use of the concept of finite yield stress at which plastic yielding begins. As a result various *ersatz* (such as energy, J-integral, stress intensity factor, fracture toughness etc. [1,2]) have been invented to circumvent the difficulty posed by the stress singularity. Clearly occurence of the singularity is a definite sign of the failure of classical elasticity in the vicinity of sharp crack tips.

Fracture of a ductile solid with crack is always accompanied by significant plastic deformations in the vicinity of crack. The plastic yielding near the crack tip, in the context of classical continuum mechanics was studied by many authors [3-9]. Celebrated among these works is the work of Bilby, Cottrell and Swinden [5] who modelled the plastic zone by an array of dislocations coplanar with the crack.

This model is adopted here with basic departure being the use of nonlocal continuum mechanics.

In material science, still other models are considered relying on the atomic and discrete granular nature of materials. Real materials, as distinguished from perfect crystals, possess extremely complicated inner structure. Mathematical analysis of crack tip problem, in the context of atomic theory, faces extreme difficulities, since the complicated, often unknown geometry of atomic distributions, large number of dislocations and impurities cannot be represented with any acuracy. Even if this were possible the atomic computations requires tedious, lengthly and high cost computations.

Recently developed nonlocal elasticity (cf. 10-14) is equipped with inner characteristic length mechanism which covers on the one hand the atomic and molecular limits on the other classical elasticity. Eringen and his co-workers have shown that Griffith's crack problems [15-18] and point dislocations [19-21] in elastic solids do not possess singular stress when they are treated within the frame of nonlocal elasticity. Moreover, fracture criteria based on the maximum stress hypothesis is valid and it predicts correct cohesive stress for perfect crystals. Several other solutions based on the nonlocal elasticity [22-24] have shown clearly the power and the potential of the nonlocal continuum mechanics.

The present work is concerned with the investigation of the plastic yield at crack tip by means nonlocal elasticty. We consider a mode III crack which containes a distributed dislocations of arbitrary magnitude and lengths along the crack line. Thus the plastic yielding envisioned is small-scale. An integral equation is obtained for the stress field along the crack line through the condition that the maximum stress reach the cohesive yield stress. Computer solution of this equation gives the dislocation density and the plastic zone size as a function of applied load.

In section 2 we present a summary of basic equations of linear, nonlocal elasticity for homogeneous, isotropic elastic solids. Section 3 contains Eringen's result [25] on the stress field due to interaction of single dislocation with crack. These results constitute the Green's function in the formulation of the distributed dislocations. Section 4 is devoted to the solution of the problem and the results obtained by computer work. Last section gives a discussion and physical significance of these calculations.

2.BASIC EQUATIONS

In several previous paper (for example cf. [26]), Eringen has shown that the field equations of linear, nonlocal elasticity for homogeneous, isotropic solid, with vanishing body forces, consist of *the equations of equilibrium*

$$t_{ij,j}(\underset{\sim}{x}) = 0 \quad , \quad t_{ij} = t_{ji} \tag{1}$$

and *the stress-strain relations*

$$t_{ij}(\underset{\sim}{x}) = {}_B\int \alpha(|\underset{\sim}{x}-\underset{\sim}{x}'|,\varepsilon)\sigma_{ij}(\underset{\sim}{x}')dv(\underset{\sim}{x}') \tag{2}$$

where σ_{ij} is the classical Hookean stress

$$\sigma_{ij}(\underset{\sim}{x}) = \lambda e_{kk}(\underset{\sim}{x})\delta_{ij} + 2\mu e_{ij}(\underset{\sim}{x}) \tag{3}$$

which is expressed as a linear function of the strain e_{ij}

$$2e_{ij}(\underset{\sim}{x}) = u_{i,j}(\underset{\sim}{x}) + u_{j,i}(\underset{\sim}{x}) \ . \tag{4}$$

Here $u_i(\underset{\sim}{x})$ is the displacement vector, λ and μ are the Lamé elastic constants and indices following comma denote the partial differentiation with respect to space variable. $\alpha(|\underset{\sim}{x}-\underset{\sim}{x}'|,\varepsilon)$ is the "*attenuation function*" which brings the effect of strains at distant points $\underset{\sim}{x}'$ in body to the stress field at a reference point $\underset{\sim}{x}$. This function depends on a characteristic length ε . The integral in (2) is over volume of the body denoted by B. The function α is a decreasing function of $|\underset{\sim}{x}-\underset{\sim}{x}'|$. Since intermolecular forces die out fast with distance, the effective range of α is of the size of molecular order. For perfect crystals it is of order of the lattice parameter while for granular and porous solids

it is of the order of micron. If we denote the external characteristic length by L and an internal characteristic length at natural state of the body by ℓ then the ratio

$$e = e_0\ell/L = \varepsilon/L \tag{5}$$

where e_0 is a dimensionless material constant, determines the range of validity of the continuum theory. For e=0 we have the classical (local) elasticity and for $e \neq 0$ nonlocal elasticity capable in dealing with microscopic and atomic phenomena. In fact it can be shown that for an appropriate function α the nonlocal elasticity gives exactly the same stress field at atomic points (cf. Eringen [27]). This function can be determined in various ways: by experiment, by statistical averages applied to atomic theories and by comparing dispersion relations of elastic waves with the phonon dispersion curves. For example, Ari and Eringen [18] using a two dimensional lattice, have obtained an excellent match in the entire Brillouin zone, with phonon dispersion curves, for

$$\alpha(|\underset{\sim}{x}|)=(2\pi\varepsilon^2)^{-1}K_o(\sqrt{\underset{\sim}{x}\cdot\underset{\sim}{x}}/\varepsilon) \tag{6}$$

where K_o is the modified Bessel's function of first kind. For other kernels in 1, 2 and 3 dimensions see [24].

It is interesting to note that the expression given by (6) is the Green's function of the linear operator $L \equiv 1-\varepsilon^2\nabla^2$ i.e.,

$$(1-\varepsilon^2\nabla^2)\alpha(|\underset{\sim}{x}-\underset{\sim}{x}'|)=\delta(|\underset{\sim}{x}-\underset{\sim}{x}'|) \tag{7}$$

where $\delta(x)$ is the Dirac delta measure. This feature of the function K_o allows us to invert the constitutive equation (2) of nonlocal elasticity:

$$(1-\varepsilon^2\nabla^2)t_{ij}=\sigma_{ij} \tag{8}$$

With this apparatus at hand, Eringen [25] gave the solution of the boundary value problem of nonlocal elasticity on the interaction of a dislocation with crack, relevant to the present work. This is summarized in the following section.

3.FORMULATION OF THE PROBLEM

The main purpose of this study is to describe the plastic zones at the tips of a crack for Mod III problem. The plastic zones will be modelled by an array of dislocations. We consider a homogeneous, isotropic, elastic medium of infinite extend weakend by a crack located at $-c \leq x_1 \leq c$, $x_2=0$, $-\infty < x_3 < \infty$ and is subject to a constant antiplane shear load τ_0 at $x_2=\mp\infty$ We assume that there exist a screw dislocation whose Burger's vector b is parallel to the x_3

axis and intersects the plane $x_3=0$ at the point S($x_1=\xi$, $x_2=0$).,(see Fig.1) . The classical solution of this problem (i.e. interaction of a crack and a dislocation) has been given by Louat[5] in a general form. The classical solution of this problem has also been given by Eringen[25] in a more appropriate form which may be expressed as follows:

$$\sigma_{23}(x_1,x_2)-i\sigma_{13}(x_1,x_2)= \\ (z^2-c^2)^{-1/2}\{\tau_0 z+(2\pi)^{-1}\mu b[1+(\xi^2-c^2)^{1/2}(z-\xi)^{-1}]\} \tag{9}$$

where $i=\sqrt{-1}$, $z=x_1+ix_2$ is the complex variable , $\bar{z}=x_1-ix_2$ being its complex conjugate. The net content of dislocations in the crack is assumed to be zero. This expression contains four different response of the medium:

(i) Stress field due to applied load $\sigma_{23} = \tau_0$

$$\sigma_{23}(x_1,x_2)-i\sigma_{13}(x_1,x_2)=\tau_0 \tag{10}$$

(ii) Stress field due to the response of the crack to the applied load:

$$\sigma_{23}(x_1,x_2)-i\sigma_{13}(x_1,x_2)=\tau_0\{\bar{z}(\bar{z}^2-c^2)^{-1/2}-1\} \tag{11}$$

(iii) Stress field due to the existence of the dislocation at $x_1=\xi$, $x_2=0$:

$$\sigma_{23}(x_1,x_2)-i\sigma_{13}(x_1,x_2)=(2\pi)^{-1}\mu b(z-\xi)^{-1} \tag{12}$$

(iv) Stress field due to the interaction of the dislocation with the crack:

$$\sigma_{23}(x_1,x_2)-i\sigma_{13}(x_1,x_2)=$$
$$(2\pi)^{-1}\mu b\{(\bar{z}^2-c^2)^{-1/2}[1+(\xi^2-c^2)^{1/2}(\bar{z}-\xi)^{-1}]-(\bar{z}-\xi)^{-1}\} \tag{13}$$

It is clear that the expression (9) has square root singularities at the crack tips and $1/r$ singularity at the dislacation. For the nonlocal solution of this problem Eringen [25] has argued that the nonlocal stress field does not possess any singularity. To obtain the nonlocal stress field it was necessary to find the solutions of the following partial differential equation

$$(1-\varepsilon^2\nabla^2)t = \sigma \tag{14}$$

where

$$t(x_1,x_2)=t_{23}(x_1,x_2)-it_{13}(x_1,x_2) \tag{15}$$

and σ is given by (9). It is easy to verify that $t=\sigma$ is a particular solution of Eq.(14). The general solution of the homogeneous equation [(14) with $\sigma=0$] that possesses proper symmetry regulations and vanishes at infinity can be formed from the generic solution

$$t = K_\nu(r/\varepsilon)\{A_\nu e^{i\nu\theta}+B_\nu e^{-i\nu\theta}\} \tag{16}$$

where A_ν , B_ν and ν are constants , $K_\nu(\rho)$ is the modified Bessel's function of first kind and (r,θ) are the plane polar coordinates.Considering the limiting behaviour of modified Bessel's functions of first kind with integer order, for small values of argument we have (c.f. (9.6.9) in [28])

$$K_\nu(z) \sim .5\Gamma(\nu)(.5z)^{-\nu} \tag{17}$$

and the special representation of $K_{1/2}$ (c.f. (10.2.17) in [28])

$$K_{1/2}(z) = \sqrt{\pi/2z}\, e^{-z} \tag{18}$$

The nonlocal stress field is then given by [25]

$$\begin{aligned} t_{23}(x_1,x_2)-it_{13}(x_1,x_2) = {} & C_1(\pi\varepsilon/2r_1)^{-1/2} e^{-r_1/\varepsilon} \; e^{-i\theta_1/2} \\ & + C_2K_1(r_3/\varepsilon)e^{i(\theta_3-\theta_1)} + (r_1r_2)^{-1/2}e^{i(\theta_2-\theta_1)/2} \{\tau_0 re^{-i\theta} \\ & + (2\pi)^{-1}\mu b\{1+[\xi(\xi+2c)]^{1/2} (r_1e^{i\theta_1}-\xi)^{-1}\} \end{aligned} \tag{19}$$

where

$$C_1=-(c/\pi\varepsilon)^{-1/2}\{\tau_0+(2\pi c)^{-1}\mu b[1-(1+2c/\xi)^{1/2}]\} \tag{20}$$

and

$$C_2=-(2\pi\varepsilon)^{-1}\mu b \tag{21}$$

For the purposes of this study we need to consider an anti-symmetric dislocation distribution. Consequently we superpose the stress field due to the dislocation located at ($x_1=-\xi$, $x_2=0$) and with -b Burger's vector to (19). With this the singularity is removed from the crack tip at x=-c . The stress distribution may be expressed as

$$t_{23}-it_{13}=(r_1r_2)^{-1/2}e^{i(\theta_1+\theta_2)/2}$$
$$\{\tau_0 re^{-i\theta}+(\pi)^{-1}\mu b[\xi(\xi+2c)]^{1/2}(\xi+c)(r_3r_4)^{-1}e^{i(\theta_3+\theta_4)}\}-$$
$$(c/2r_1)^{1/2}\{\tau_0-(\pi c)^{-1}\mu b[\xi(\xi+2c)]^{-1/2}(\xi+c)\}e^{-r_1/\varepsilon}\ e^{i\theta_1/2}-$$
$$(c/2r_2)^{1/2}\{\tau_0+(\pi c)^{-1}\mu b[\xi(\xi+2c)]^{-1/2}(\xi+c)\}e^{-r_2/\varepsilon}\ e^{-i\theta_2/2}-$$
$$(2\pi\varepsilon)^{-1}\mu b\{e^{i\theta_3}K_1(r_3/\varepsilon)+e^{i\theta_4}K_1(r_4/\varepsilon)\} \qquad (22)$$

where θ_i and r_i are the polar coordinates as shown in Fig.2 . Since we are dealing with small scale yielding near crack tip we need only the t_{23} component of stress on the plane coplanar with the crack. From (22) we have

$$t_{23}(x,0)=t^c(x)+t^{cd}(x)$$
$$t^c(x)=\tau_0\{[x(x+2c)]^{-1/2}(x+c)-(c/2x)^{1/2}e^{-x/\varepsilon}\}$$
$$t^{cd}(x,\xi)=(\pi^{-1}\mu)$$
$$\{[x(x+2c)]^{-1/2}[\xi(\xi+2c)]^{1/2}(x-\xi)^{-1}(x+\xi+2c)^{-1}(\xi+c)+$$
$$(2c)^{-1/2}[\xi(\xi+2c)]^{-1/2}(\xi+c)x^{-1/2}e^{-x/\varepsilon}-$$
$$(2\varepsilon)^{-1}[\operatorname{sgn}(x-\xi)K_1(|x-\xi|/\varepsilon)+K_1((x+\xi+2c)/\varepsilon)]\}$$

(23)

where x denotes the distance from the crack tip at right hand side. The stress field, due to an array of dislocations in the interval (a,b) with density $b(\xi)$ can now be expressed as:

$$t(x) = t^{c}(x) + \int_a^b t^{cd}(x,\xi)b(\xi)d\xi \qquad (24)$$

4.SOLUTION OF THE PROBLEM

From the point of our intrest we aim to solve the Eq.(24) for $t(x)=t_y$, $d<x<e$, where t_y is the cohesive stress between atoms and d,e are the coordinates of the plastic zone. The equation which we need to solve is a Fredholm integral equation of first kind with a smooth kernel. The well-known ill-posed character of such equations increases with the smoothness of the kernel. In general, all information provided for these equations, either analytical or numerical point of view, is crude rule of thumb rather than a precise guide to the nature of the problem. For analytical and numerical treatment of Fredholm integral equation of first kind [29-32] can be referred.

Among the numerical techniques for solving the Fredholm integral equation of first kind, the so-called *Minimal Least Square Solution* is perhaps the most reliable and handy one. According to this method, the unknown function is approximated by a set of basis function

$$b(\xi) = \sum_{i=1}^{n} b_i \ \phi_i(\xi) \qquad (25)$$

and the unknown coefficients b_i are so described that the difference between the results obtained from the integral with (25) and the desired result will be minimum with respect to a norm

$$\left\| \sum_{i=1}^{n} b_i \int_a^b t^{cd}(x,\xi)\phi_i(\xi)d\xi + t^c(x) - t_y \right\| \to \min \quad d<x<e \tag{26}$$

In some cases, some restrictions on b_i may accompany to this minimization problem.

As a first approach

$$\phi_i(\xi) = 1 \qquad \xi_i \le \xi \le \xi_{i+1} \tag{27}$$

can be chosen. Here ξ_i and ξ_{i+1} denote the coordinates of a subinterval in (a,b). With this set of basis function it can be constructed a simple algorithm since $t^{cd}(x,\xi)$ can be integrated with respect to ξ :

$$\Psi_i(x) = \int_{\xi_i}^{\xi_{i+1}} t^{cd}(x,\xi)d\xi \tag{28}$$

Before performing the integration let us normalize the distances with the half crack length c and the stresses with the cohesive stress t_y

$$z = x/c \quad , \quad \eta = \xi/c \quad , \quad R = \tau_o/t_y \tag{29}$$

In terms of these new variable we have

$$t^{c}(z)=R\{[z(z+2)]^{-1/2}(z+1)-(2z)^{-1/2}e^{-(c/\varepsilon)z}\}$$

$$t^{cd}(z,\eta)=(\pi c)^{-1}\mu$$

$$\{[z(z+2)]^{-1/2}[\eta(\eta+2)]^{1/2}(z-\eta)^{-1}(z+\eta+2)^{-1}(\eta+1)+$$

$$(1/\sqrt{2})[\eta(\eta+2)]^{-1/2}(\eta+1)z^{-1/2}e^{-(c/\varepsilon)z}-$$

$$(c/2\varepsilon)\{\mathrm{sgn}(z-\eta)K_1[(c/\varepsilon)|z-\eta|]+K_1[(c/\varepsilon)(x+\eta+2)]\}\}$$

$$(30)$$

To calculate the integral (28) we consider the following indefinite integrals:

$$\int[x(x+2)]^{1/2}(y-x)^{-1}dx=-[x(x+2)]^{1/2}+[y(y+2)]^{1/2}$$

$$\{\ln\{[y(y+2)]^{1/2}[x(x+2)]^{1/2}-(y+1)(y-x)+y(y+2)\}-$$

$$\ln(y-x)\}+(z+1)\ln\{[x(x+2)]+(y-x)-(y+1)\} \quad (31)$$

It should be noted that the definite value of this integral exists in the sense of Cauchy Principle Value if z takes place in the integration interval.

$$\int[x(x+2)]^{1/2}(y+x+2)^{-1}dx=-[x(x+2)]^{1/2}+[y(y+2)]^{1/2}$$

$$\{\ln\{[y(y+2)]^{1/2}[x(x+2)]^{1/2}-(y+1)(y+x+2)+y(y+2)\}-$$

$$\ln(y+x+2)\}+(z+1)\ln\{[x(x+2)]+(y+x+2)-(y+1)\} \quad (32)$$

$$\int[x(x+2)]^{-1/2}(x+1)dx=[x(x+2)]^{1/2} \quad (33)$$

$$\int\mathrm{sgn}(y-x)K_1(|y-x|)dx=K_o(|y-x|) \quad (34)$$

Also the definite value of this integral exists in the sense of Cauchy Principle Value if z is in the integration interval. With the aid of these results $\Psi_i(z)$ can be expressed as follows:

$$\begin{aligned}\Psi_i(z)=&(\pi^{-1}\mu)\{(2/A)(R_2-R_1)+\ln E_1+\ln E_2+\\&(R_2-R_1)(2z)^{-1/2}e^{-(c/\varepsilon)z}-\\&(1/2)[K_o((c/\varepsilon)|A_1|)-K_o((c/\varepsilon)|A_2|)-\\&\quad K_o((c/\varepsilon)(B+B_1))+K_o((c/\varepsilon)(B+B_2))]\end{aligned} \tag{35}$$

where

$$\begin{aligned}&\eta_i=c^{-1}[a+(b-a)(i-1)/N] \qquad i=1,2,\ldots,N+1\\&A=[z(z+2)]^{1/2}\ ,\quad A_1=z-\eta_{i+1}\ ,\quad A_2=z-\eta_i\\&B=z+1\ ,\quad B_1=\eta_{i+1}+1\ ,\quad B_2=\eta_i+1\\&R_1=[\eta_{i+1}(\eta_{i+1}+2)]^{1/2}\ ,\quad R_2=[\eta_i(\eta_i+2)]^{1/2}\\&E_1=|A_2/A_1|(B+B_1)^{-1}(B+B_2)\\&E_{21}=(A.R_1-B.A_1+A^2)(A.R_1-B.B_1-1)\\&E_{22}=(A.R_2-B.A_2+A^2)(A.R_2-B.B_2-1)\\&E_2=E_{21}/E_{22}\end{aligned} \tag{36}$$

Let us define *the residuals*

$$r_j = \sum_{i=1}^{n} b_i\,\Psi_i(z_j) + t^C(z_j) - t_y \qquad d<z_j<e \tag{37}$$

Now, we wish to describe the unknown coefficients b_i by minimizing the squares of residuals

$$F = \sum_{j=1}^{m} (r_j)^2 \tag{38}$$

It should be noted that a necessary condition for the existence of a set of b_i's which minimize the functional F is that $m > n$. The fundamentals of the solution techniques of these kind of problems Dennis and Schnabel [33] can be consulted. Since the experimental results [34] indicate that the dislocations near a creack tip have always the same sign we solve the Eq.(38) with the constrains

$$b_i \geq 0 \tag{39}$$

In the solution of this problem the following points are important:

(i) The begining and end points of the dislocation zone (a,b) and of the plastic zone (d,e) are the unknowns of the problem for a given applied load. There is no appearent mathematical reason to take a=d , b=e which is the case in classical approach. Neverthless, good results (in stress and dislocation distribution) are obtained when they are chosen very close to each other.

(ii) An important difference between classical and nonlocal approach is the occurence of the plastic yield at crack tip. As is well known, in classical approach, plastic yielding occures for every value of applied load. But this is not true in nonlocal approach. Since the response of a crack to

the external load possesses an extremum near the crack tip in nonlocal approach the plastic yielding will begin after a definite value of applied load for which the extremum value of elastic response exceeds a prescribed limit (maximum stress hypothesis). So, the nonlocal elasticity enables us to define a critical value of applied load after which the plastic yielding occures at the crack tips.

(iii) The coordinates of dislocation zone (or plastic zone, assuming the coordinates of both zones are same) are unknown. An examination of the behaviour of the nonlocal elastic stress field near crack tip clearly indicates that the beginning point of dislocation zone is very close to the point at which the elastic stress distribution reaches its extremum value. The end point of dislocation zone is a function of the applied load. To determine this point for a given value of applied load it is necessary to solve a difficult nonlinear problem in which applied load is unknown. Instead of tedious computations required by this process we choose the applied load as unknown for a given value of the end coordinate of plastic zone. This approach provides much simplicity and economy in the volume of computations.

5.RESULTS AND DISCUSSION

The problem to be solved is to determine the dislocation densities b_i and the applied load τ_0 so that the functional F given by (38) reaches its minimum value under the constrains (39). The main parameters of the problem are the internal characteristic length (ε) and the half crack length (c). The results obtained for ε = .00001(cm.) and c = .001(cm.) are given in a series of figures (Fig.3-10). In the figures marked with (a) are shown the stress distribution. The curves marked as "elastic" and "interaction" show the contribution of the term t^c and t^{cd} which are given by (30), respectively. The curves marked as "elasto- plastic" show the totality of these terms. As is seen clearly from these figures the behaviour of elasto-plastic stress field is quite reasonable; stress increases to a definite value (t_y), stays unchanged (plastic zone) and decreases to another value (applied load) asymptotically. The increasing and decreasing parts of these curves are the elastic response of the medium. Another important aspect is "dislocation free zone" which is clearly seen in the figures marked with (b) in which the dislocation distribution are shown. Dislocation free zone which is observed by in-situ electron microscope experiments [34] is a natural result of nonlocal approach i.e. needs no further assumption. In another series of figures (Fig.11-13) the results obtained for ε =

.00001(cm.) and c = .1(cm.) are given. The plastic zone is quite small as compared with the half crack length, in these figures. For larger plastic zone it is neccesary to take large number of division of the dislocation zone which makes computation cost high. If the division number is not sufficiently large then approximation is not good enough as can be observed from Fig.13 . In the last figure (Fig.14) the end coordinate of plastic zone obtained by nonlocal and classical approach are shown. The nonlocal result is remarkabely smaller than the classical result. But this is quite reasonable because there is no plastic yielding for some values of applied load smaler than a definite limit in nonlocal approach while classical approach gives some plastic zone length.

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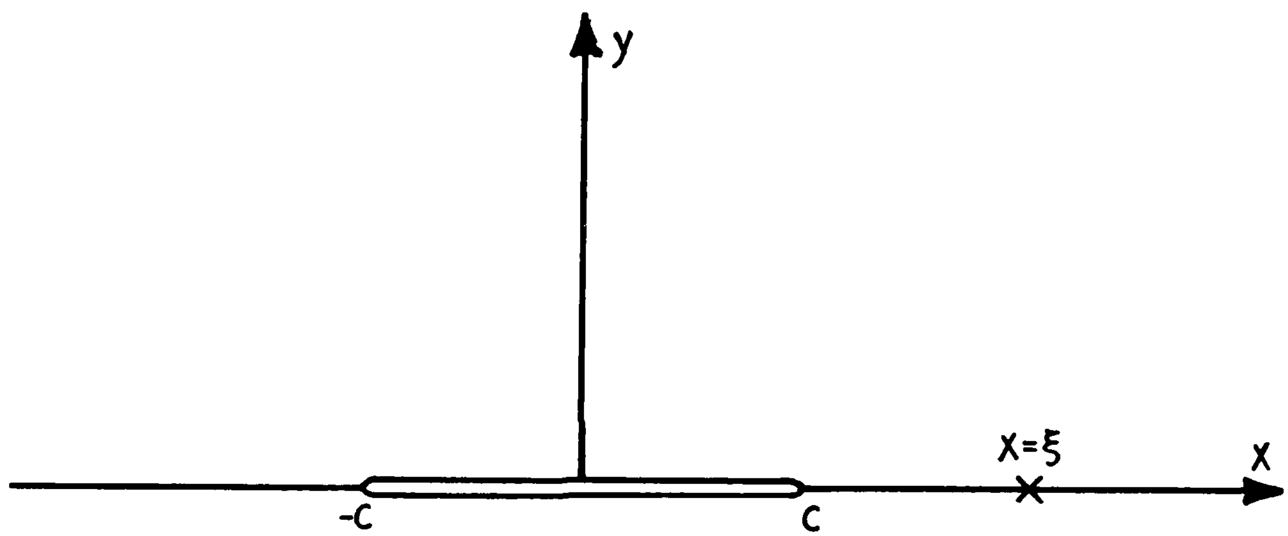

Fig. 1. Mode III Crack and Dislocation

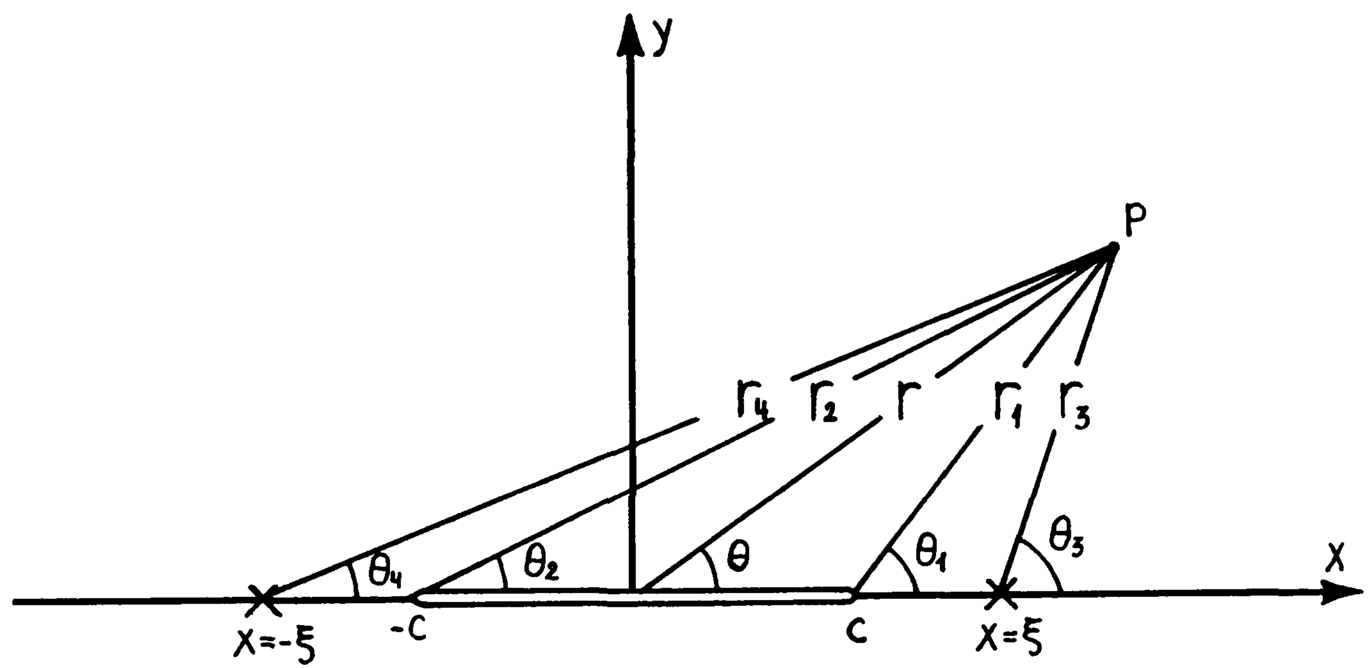

Fig. 2. Corrdinates for Crack-Dislocation Interaction

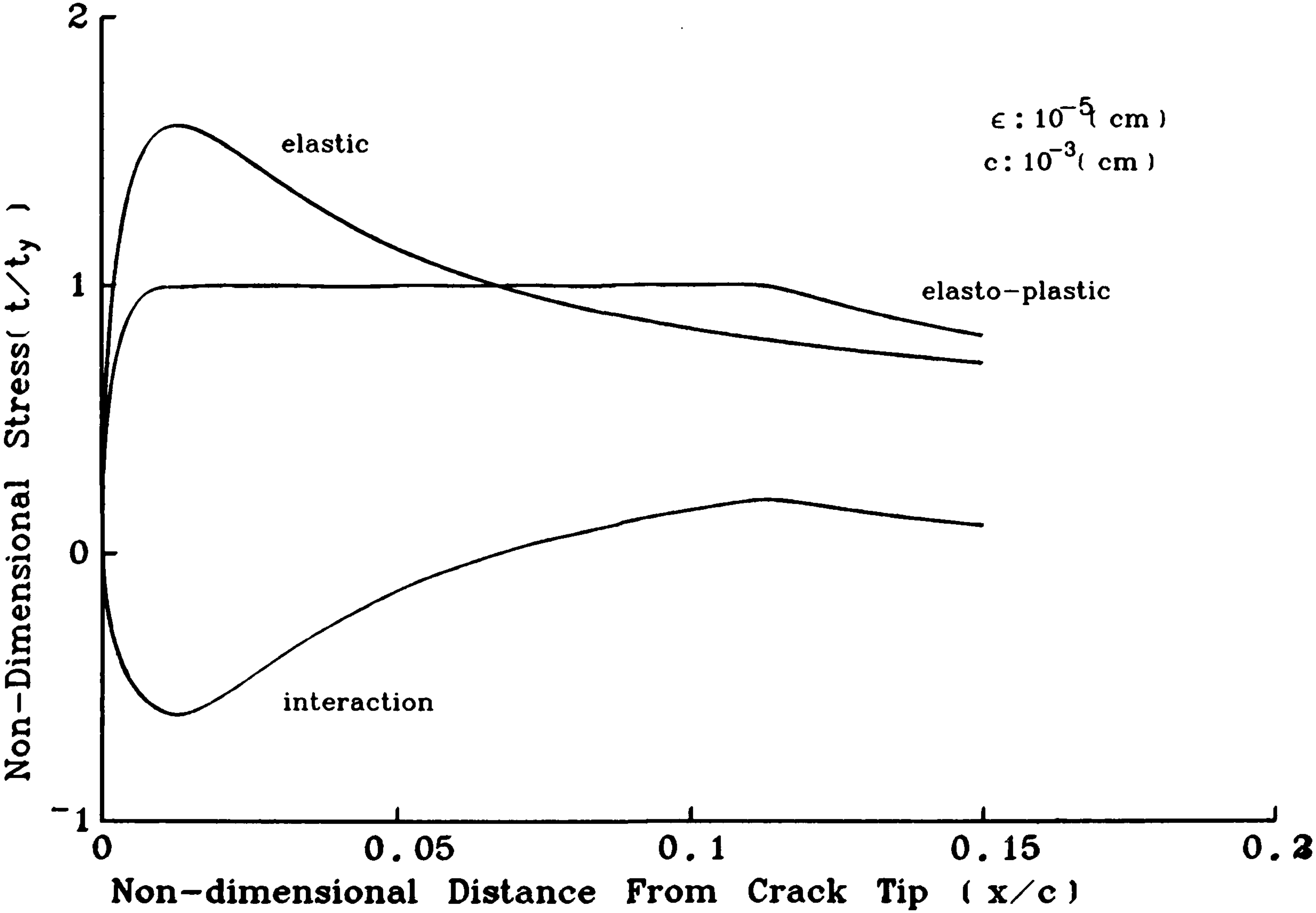

Fig. 3a. Stress Distribution Near Crack Tip (R = 0.349)

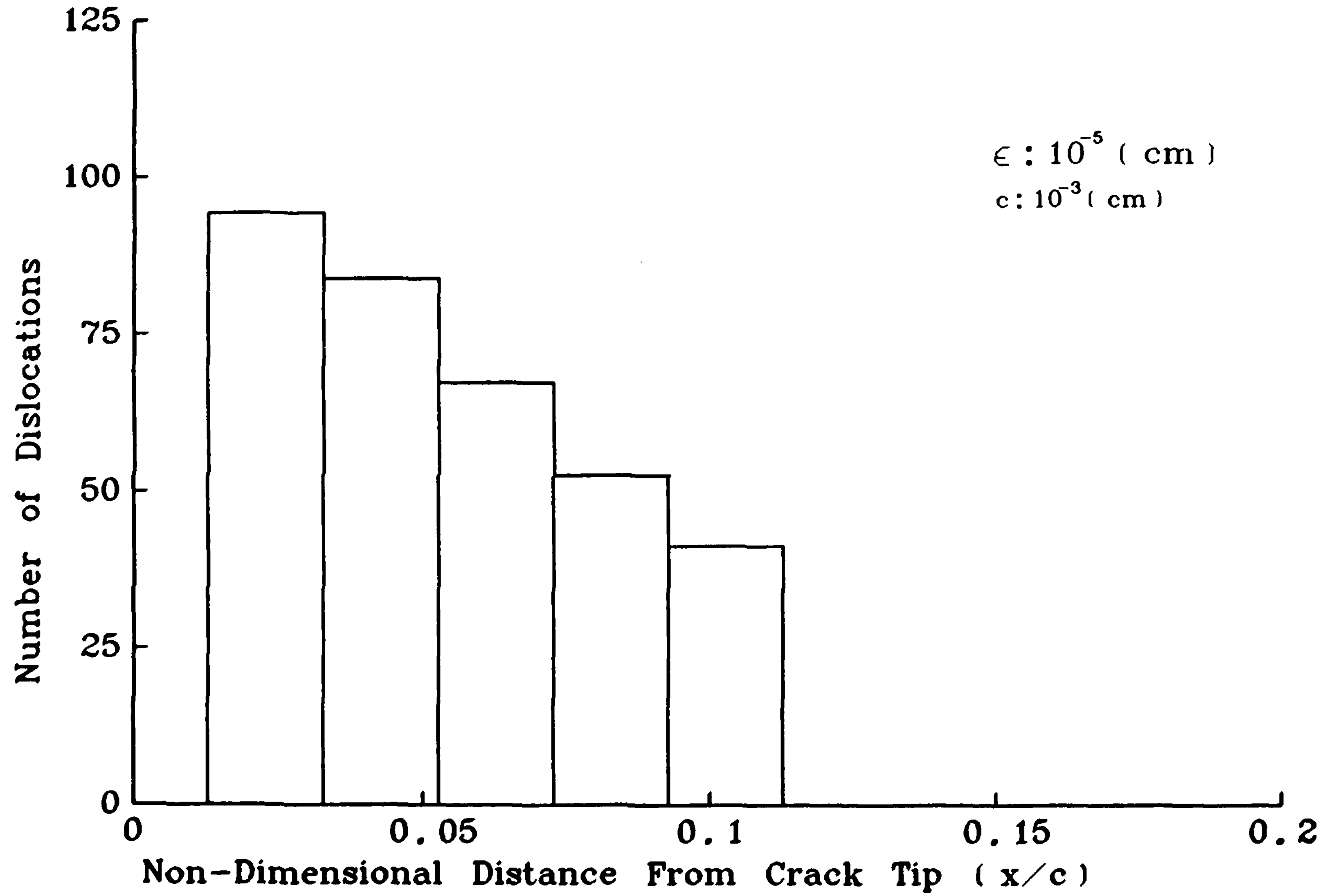

Fig. 3b. Dislocation Distribution Near Crack Tip (R = 0.349)

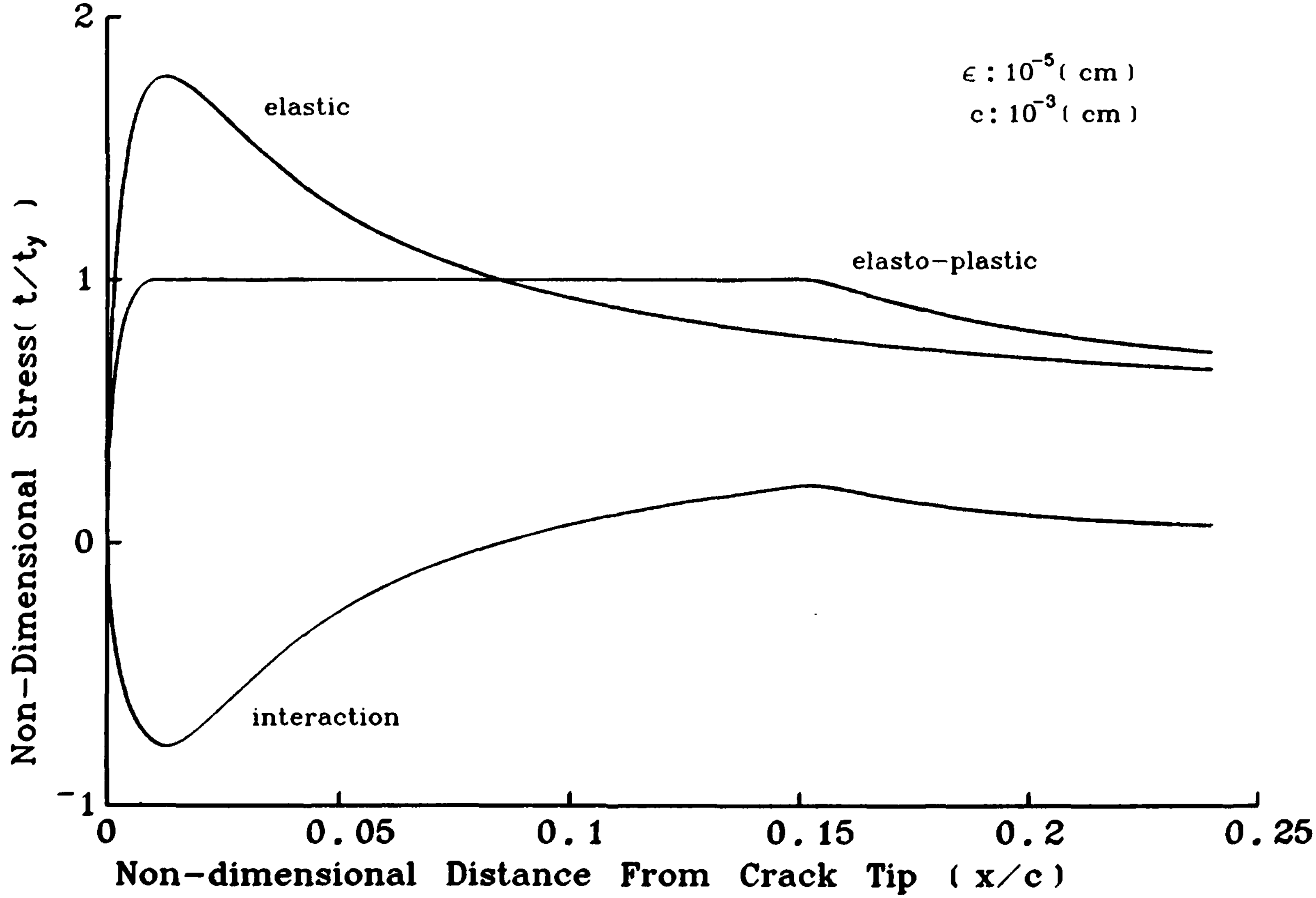

Fig. 4a. Stress Distribution Near Crack Tip (R = 0.388)

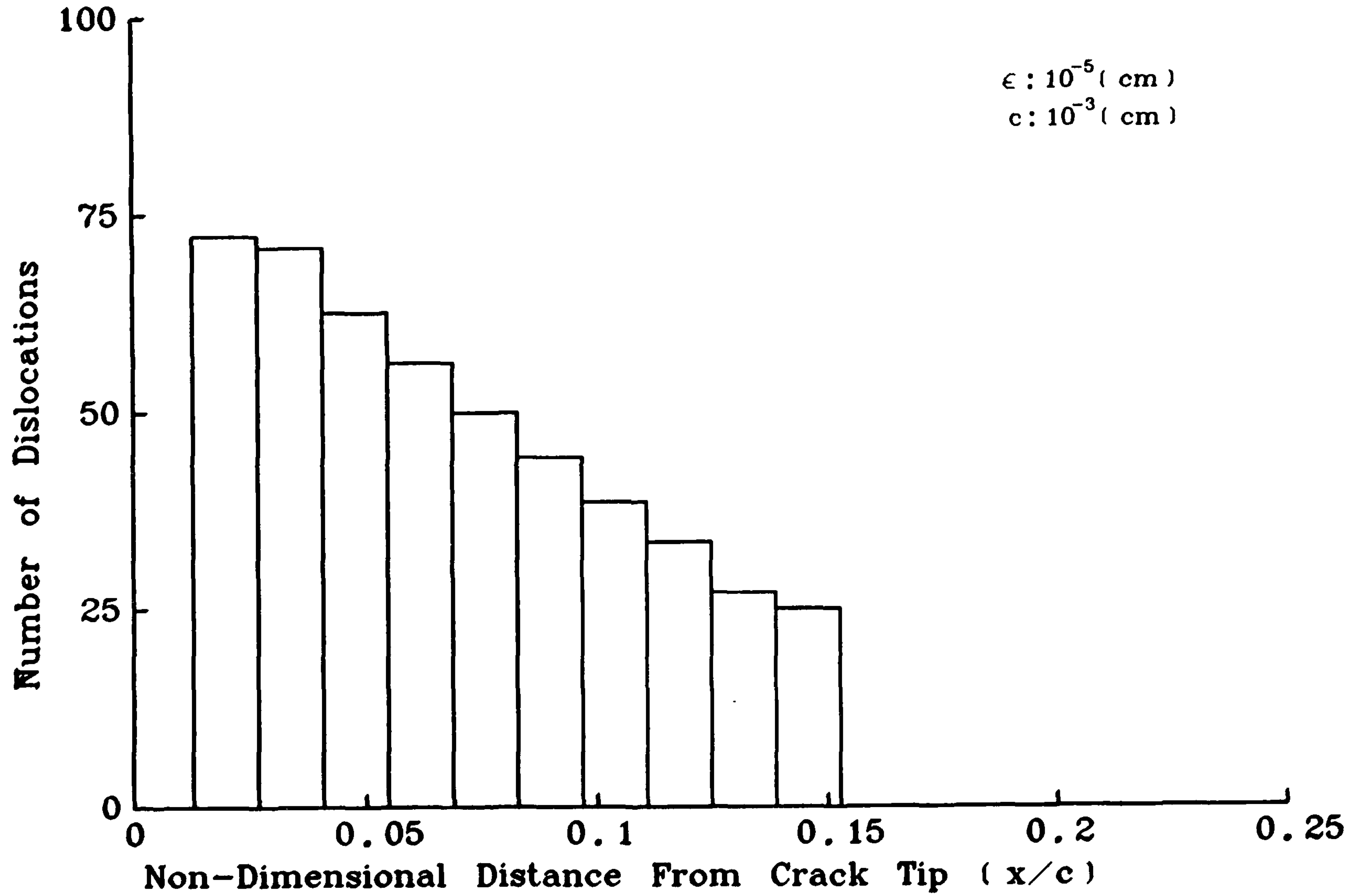

Fig. 4b. Dislocation Distribution Near Crack Tip (R = 0.388)

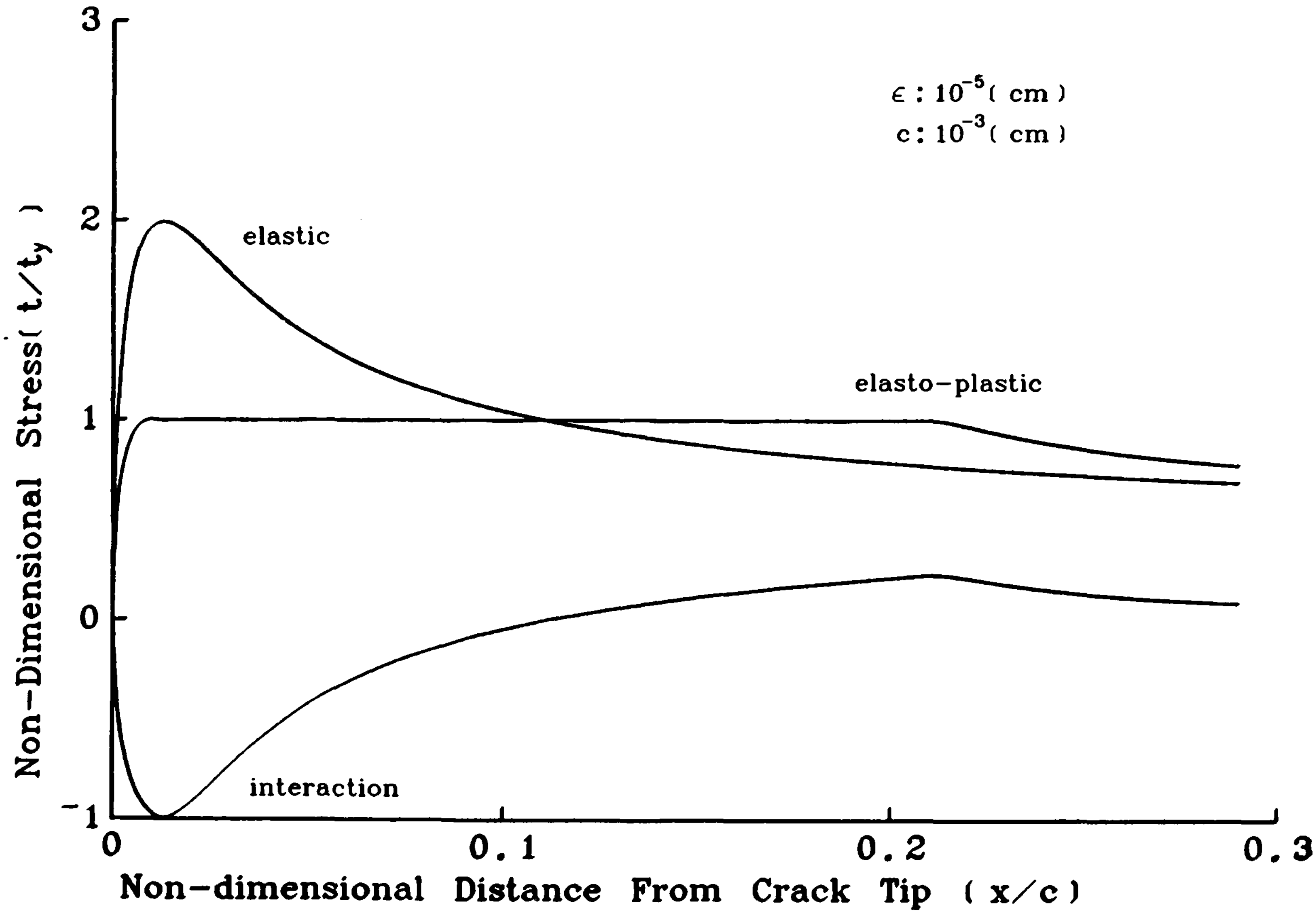

Fig. 5a. Stress Distribution Near Crack Tip (R = 0.436)

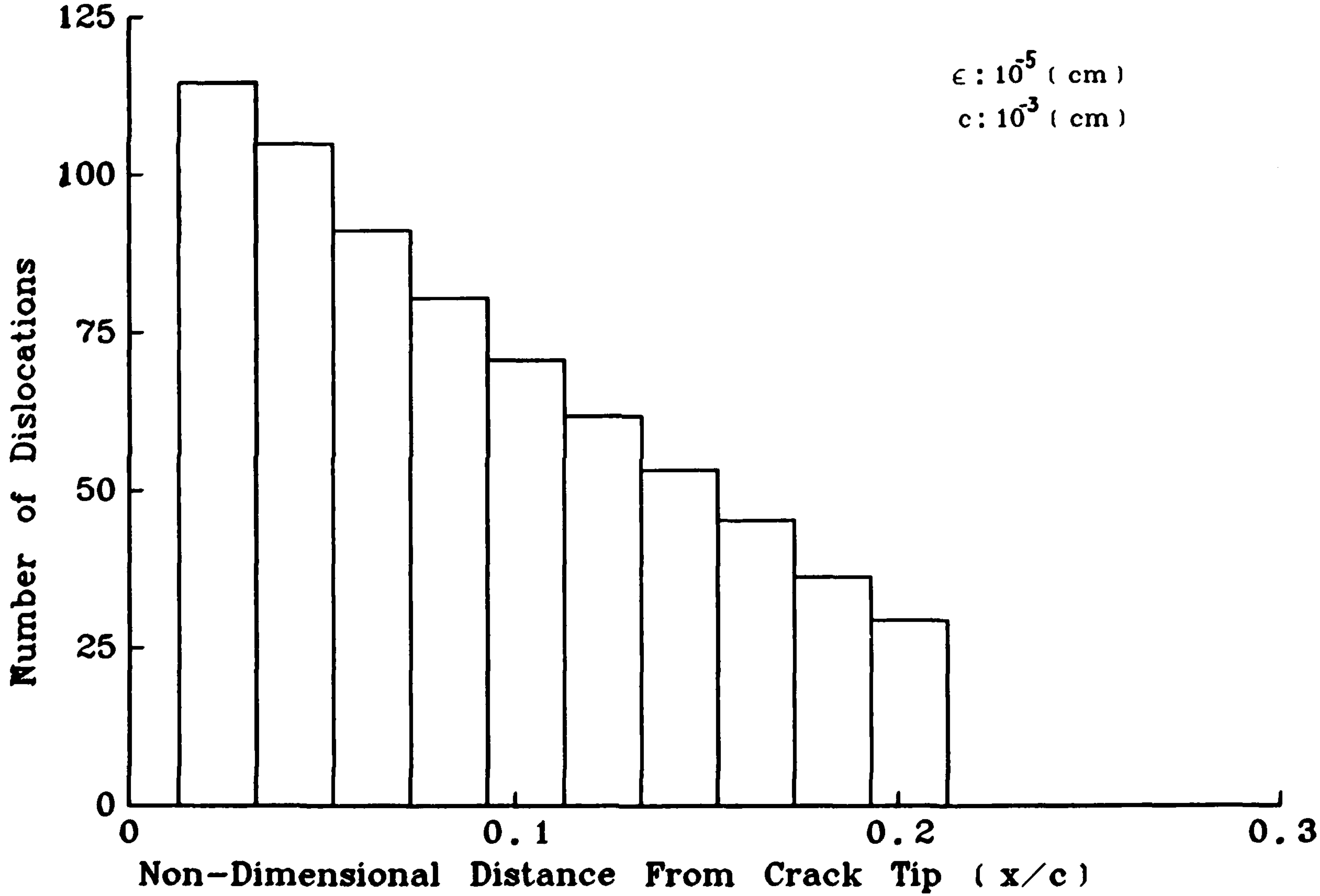

Fig. 5b. Dislocation Distribution Near Crack Tip (R = 0.436)

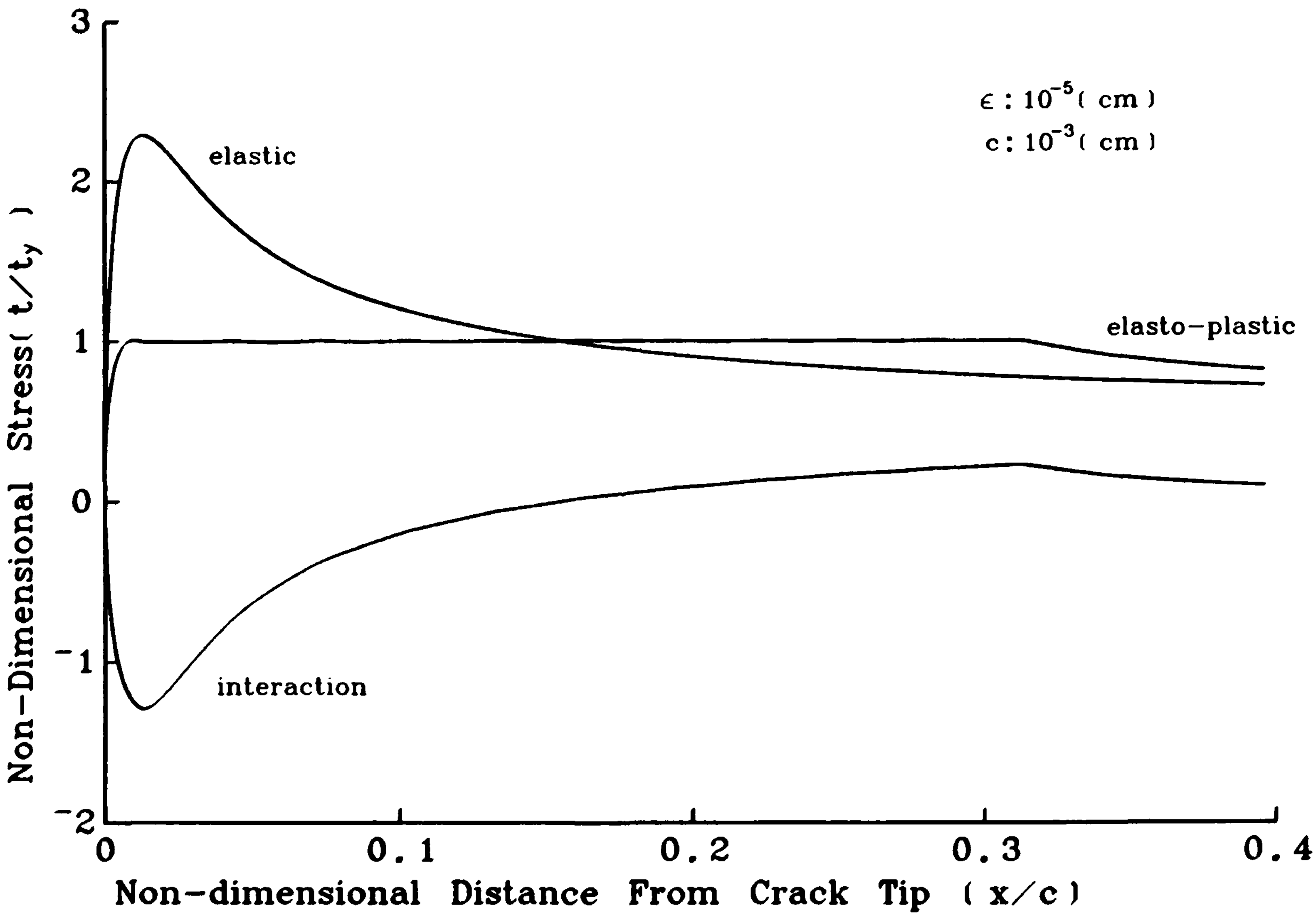

Fig. 6a. Stress Distribution Near Crack Tip (R = 0.500)

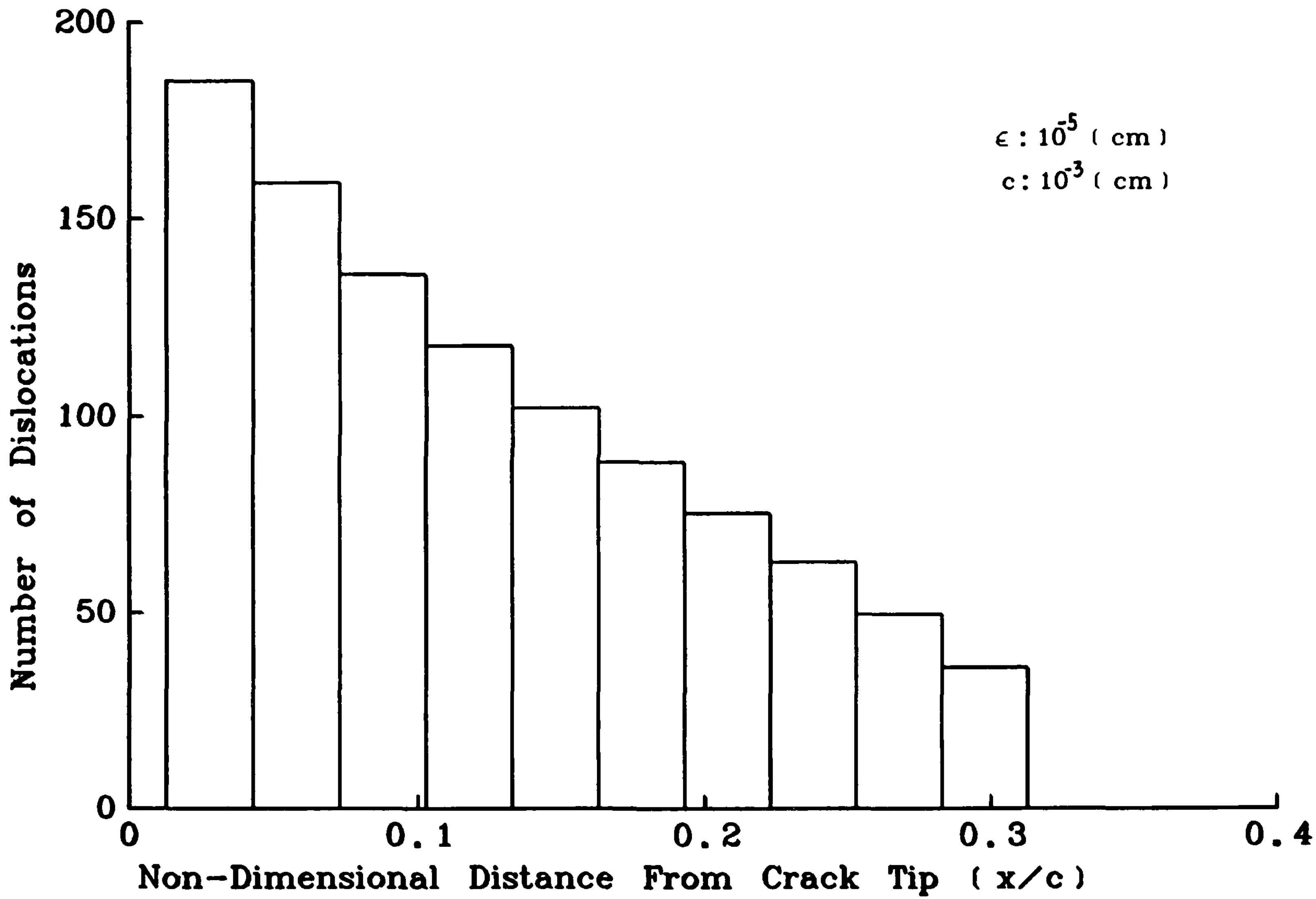

Fig. 6b. Dislocation Distribution Near Crack Tip (R = 0.500)

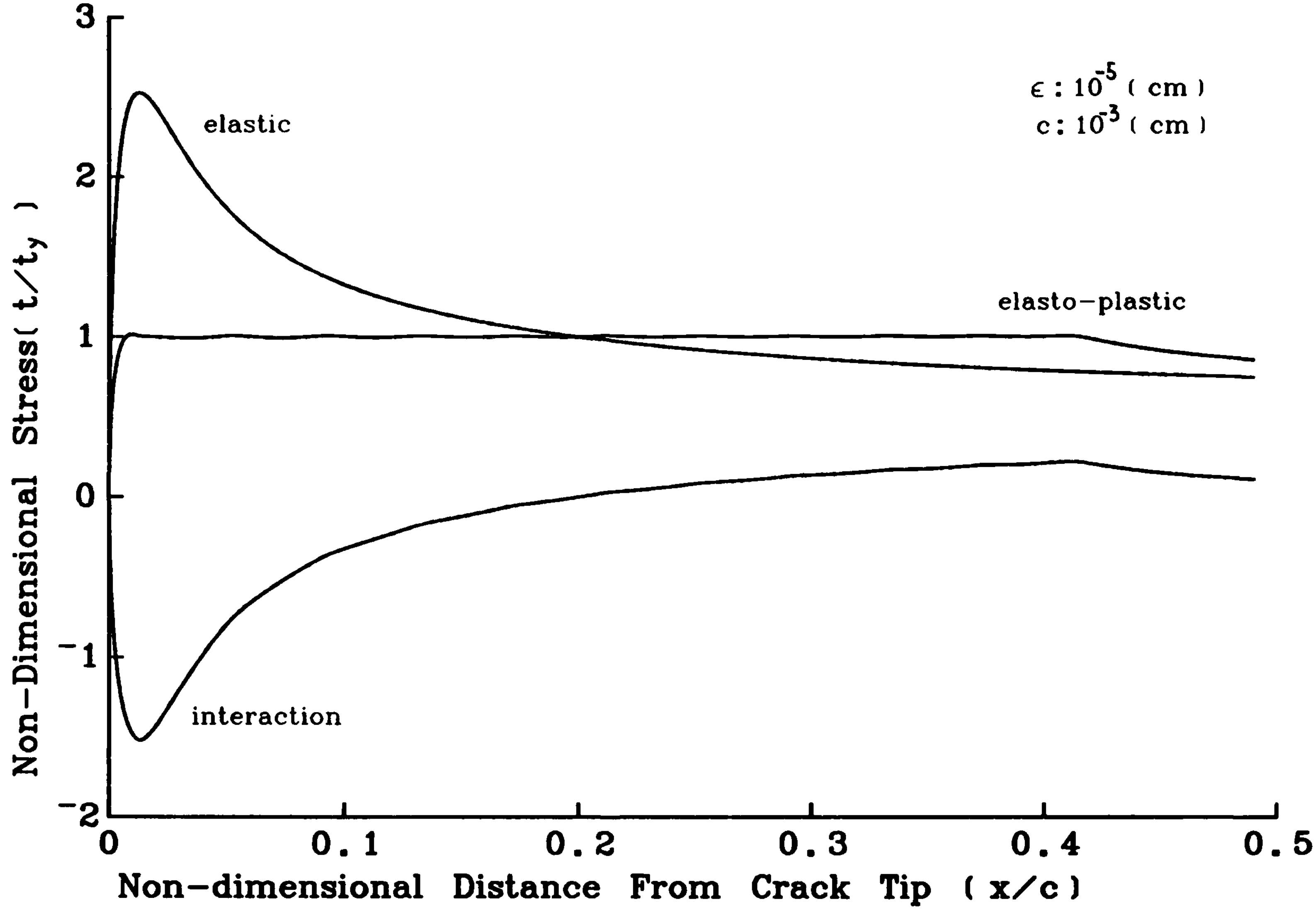

Fig. 7a. Stress Distribution Near Crack Tip (R = 0.552)

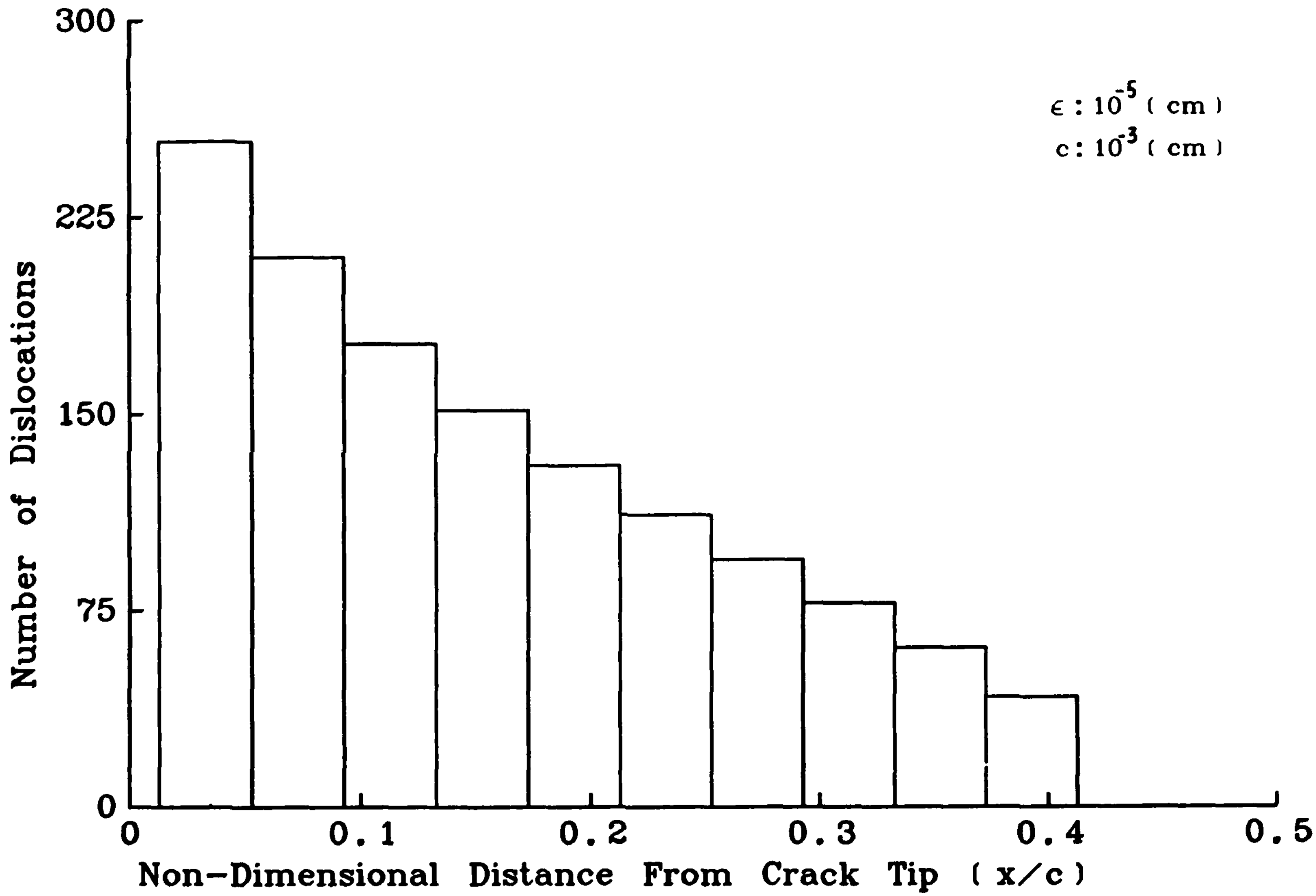

Fig. 7b. Dislocation Distribution Near Crack Tip (R = 0.552)

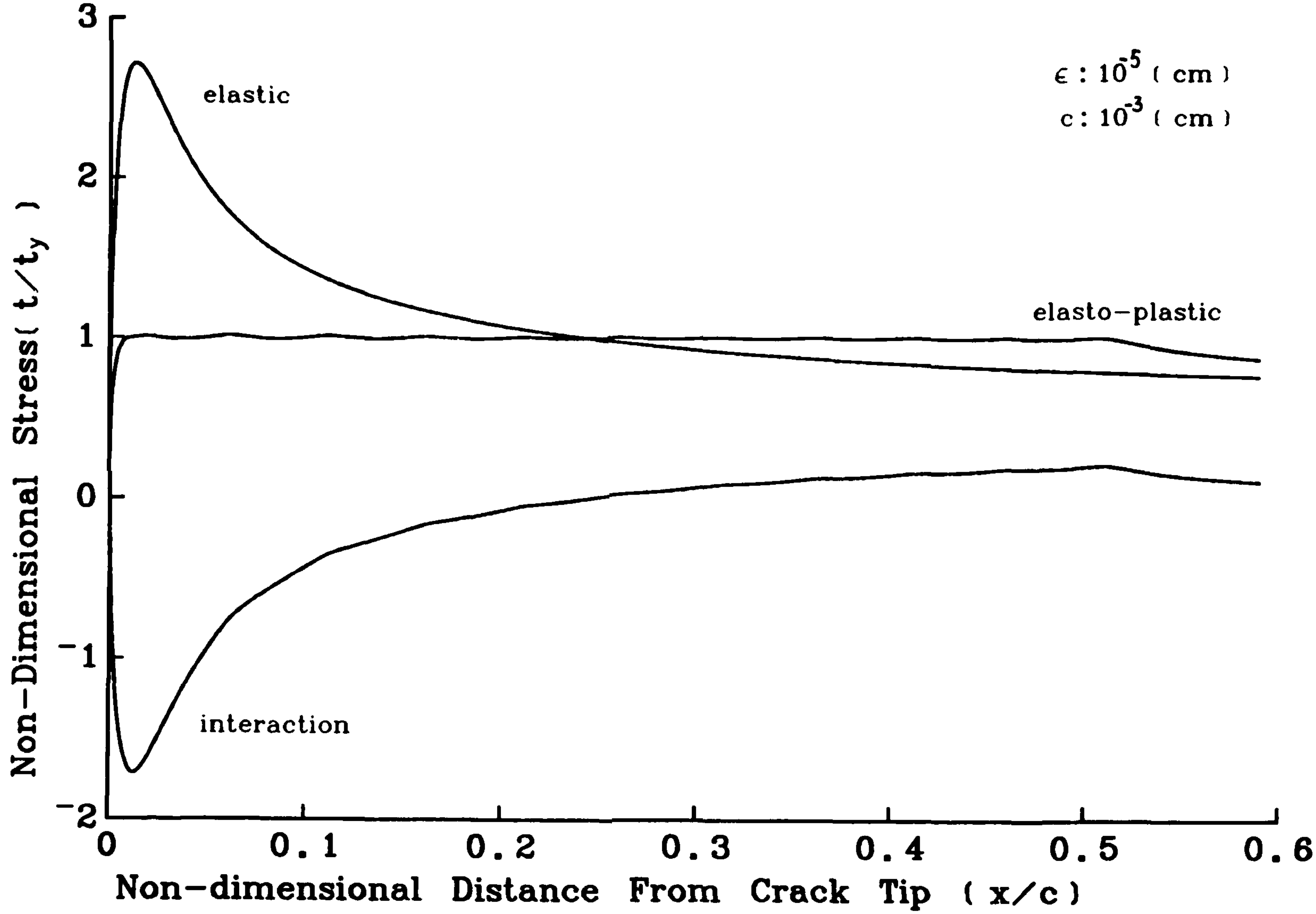

Fig. 8a. Stress Distribution Near Crack Tip (R = 0.594)

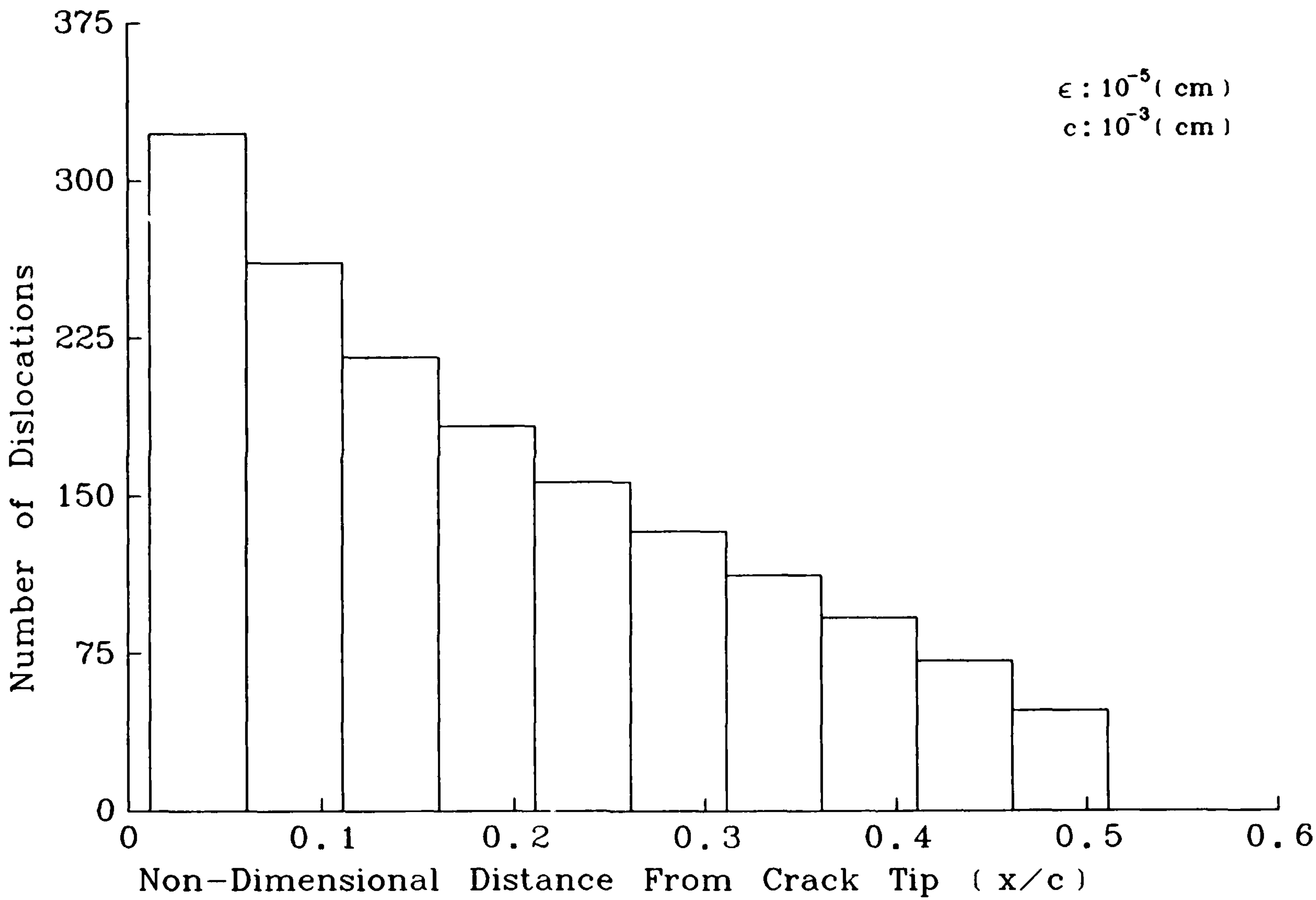

Fig. 8b. Dislocation Distribution Near Crack Tip (R = 0.594)

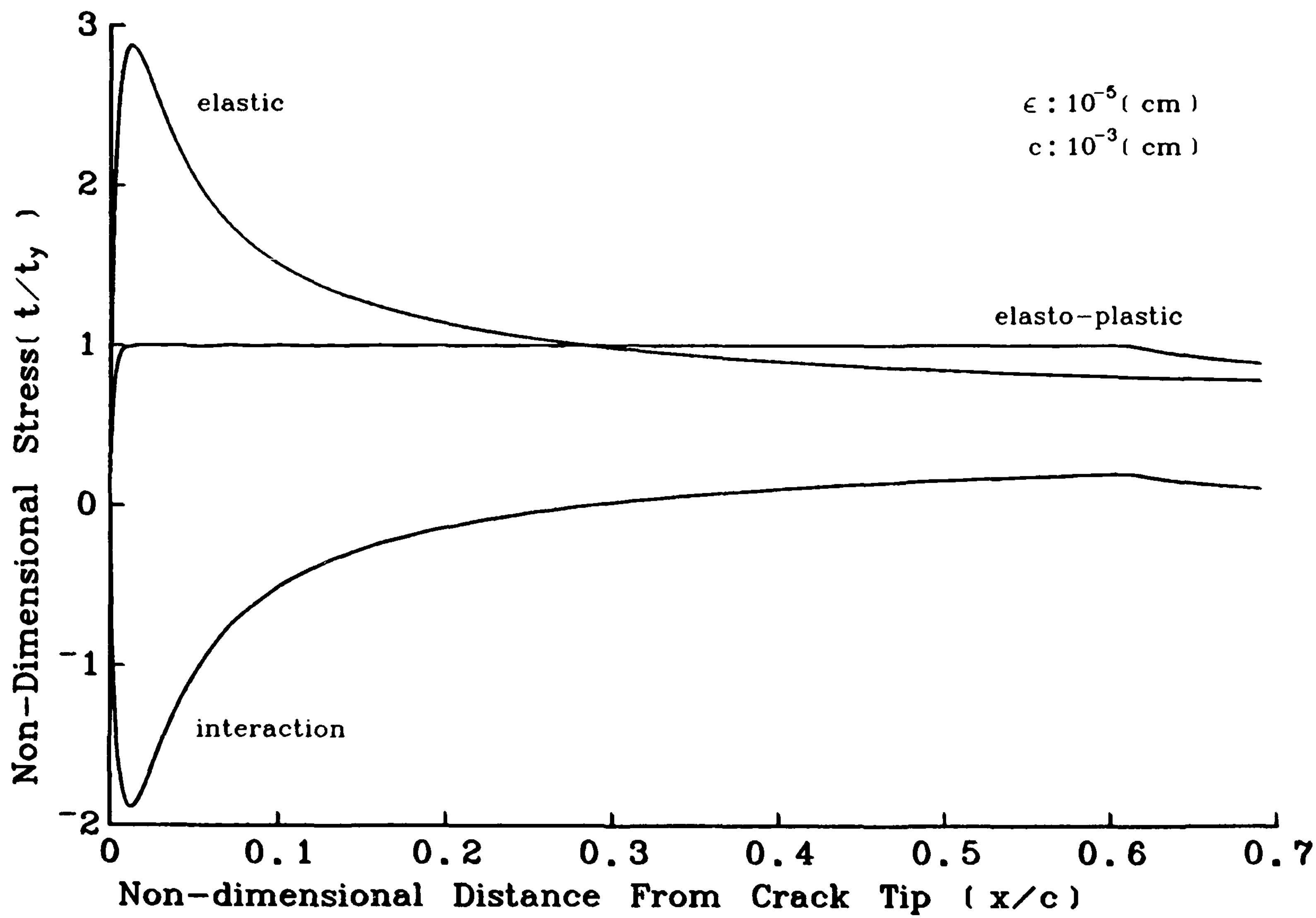

Fig. 9a. Stress Distribution Near Crack Tip (R = 0.629)

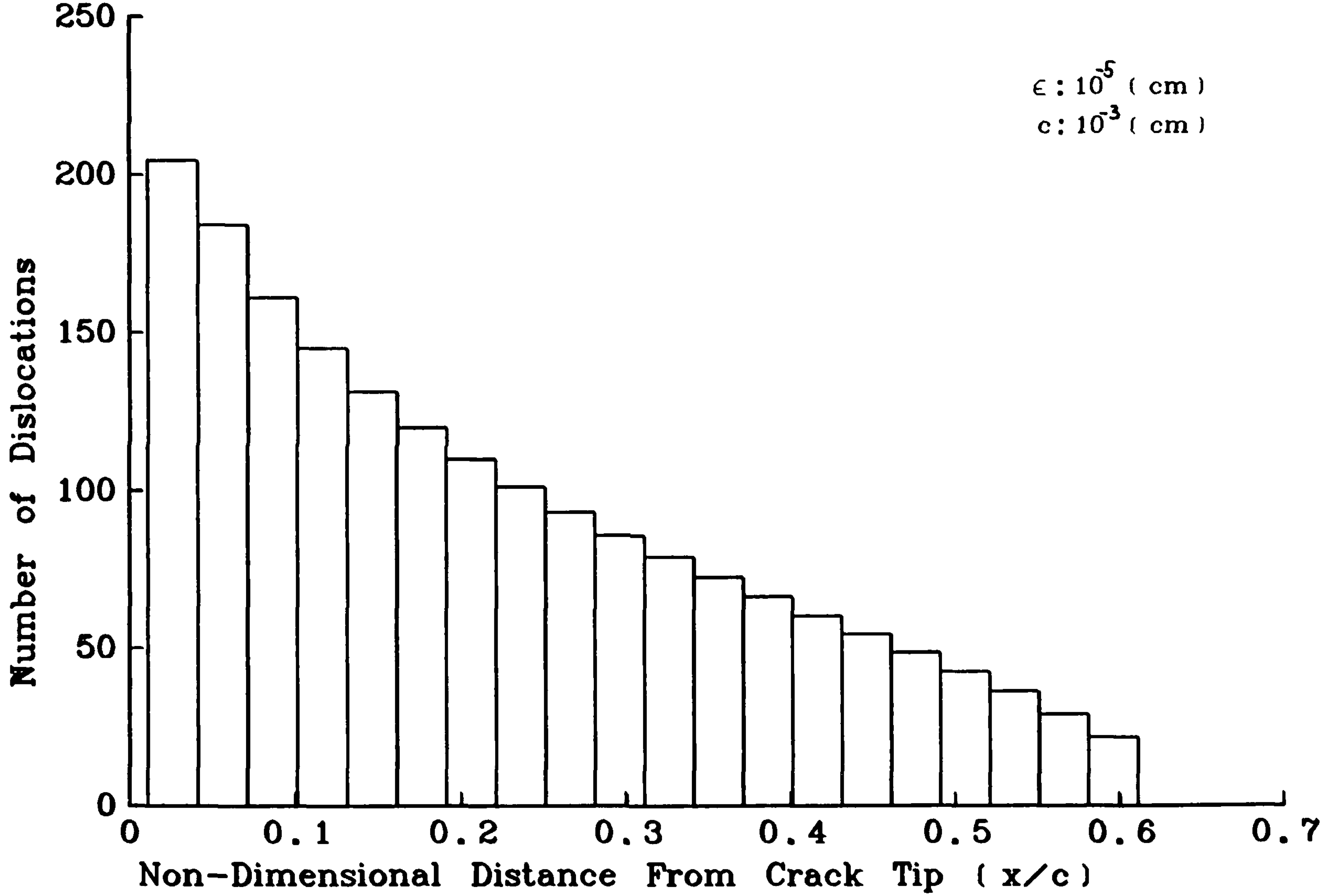

Fig. 9b. Dislocation Distribution Near Crack Tip (R = 0.629)

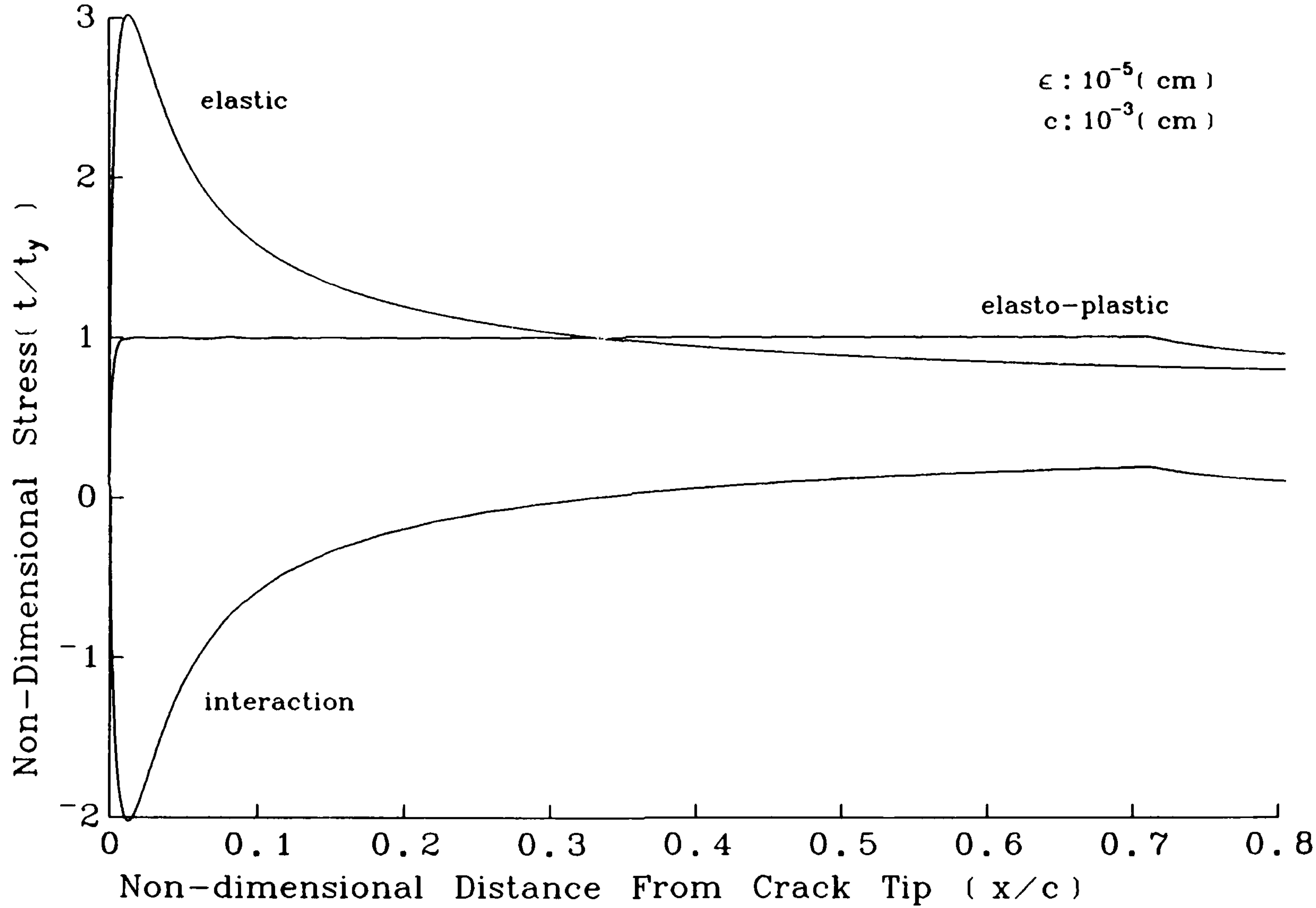

Fig. 10a. Stress Distribution Near Crack Tip (R = 0.660)

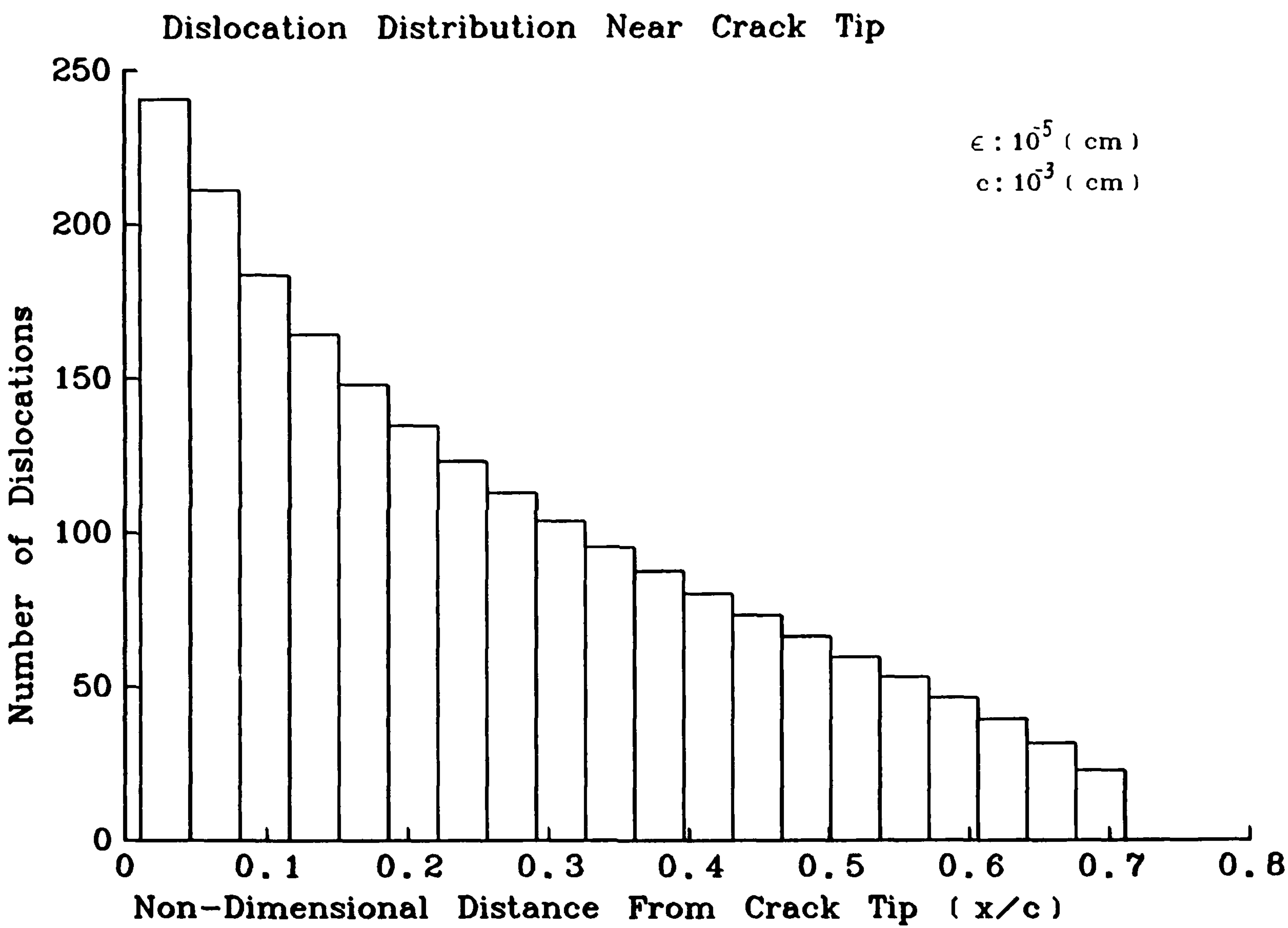

Fig. 10b. Dislocation Distribution Near Crack Tip (R = 0.660)

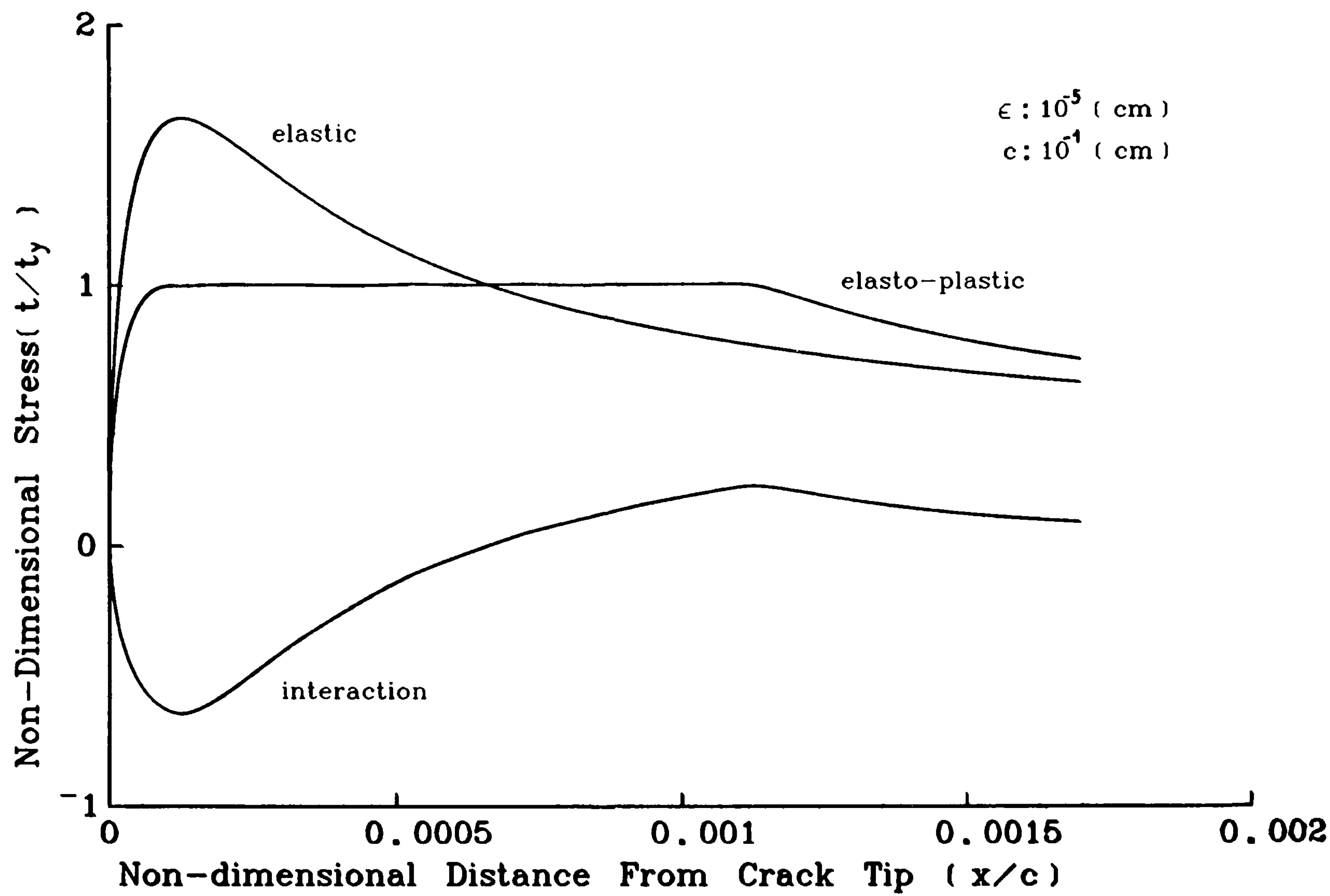

Fig. 11a. Stress Distribution Near Crack Tip (R = 0.036)

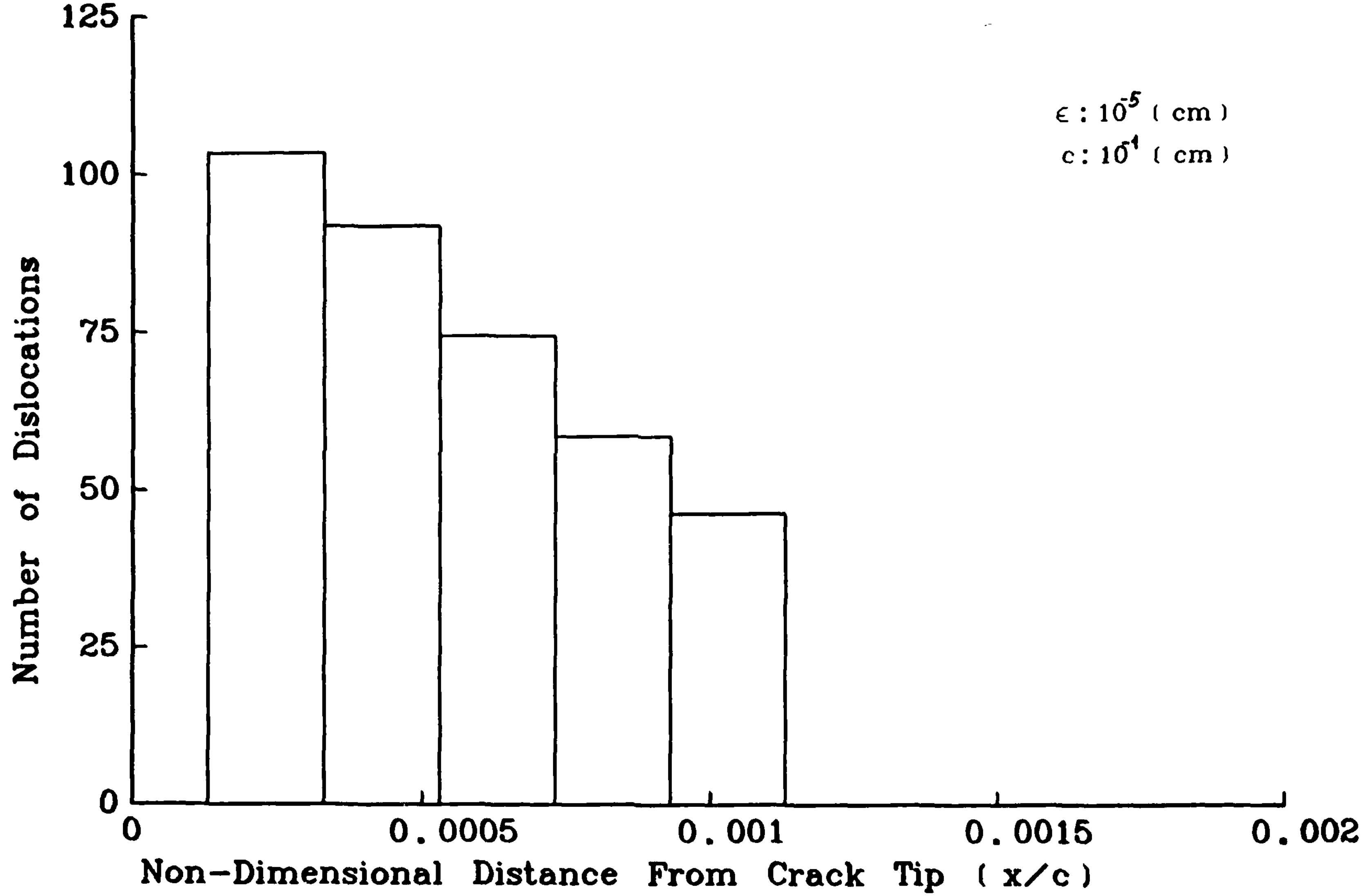

Fig. 11b. Dislocation Distribution Near Crack Tip (R = 0.036)

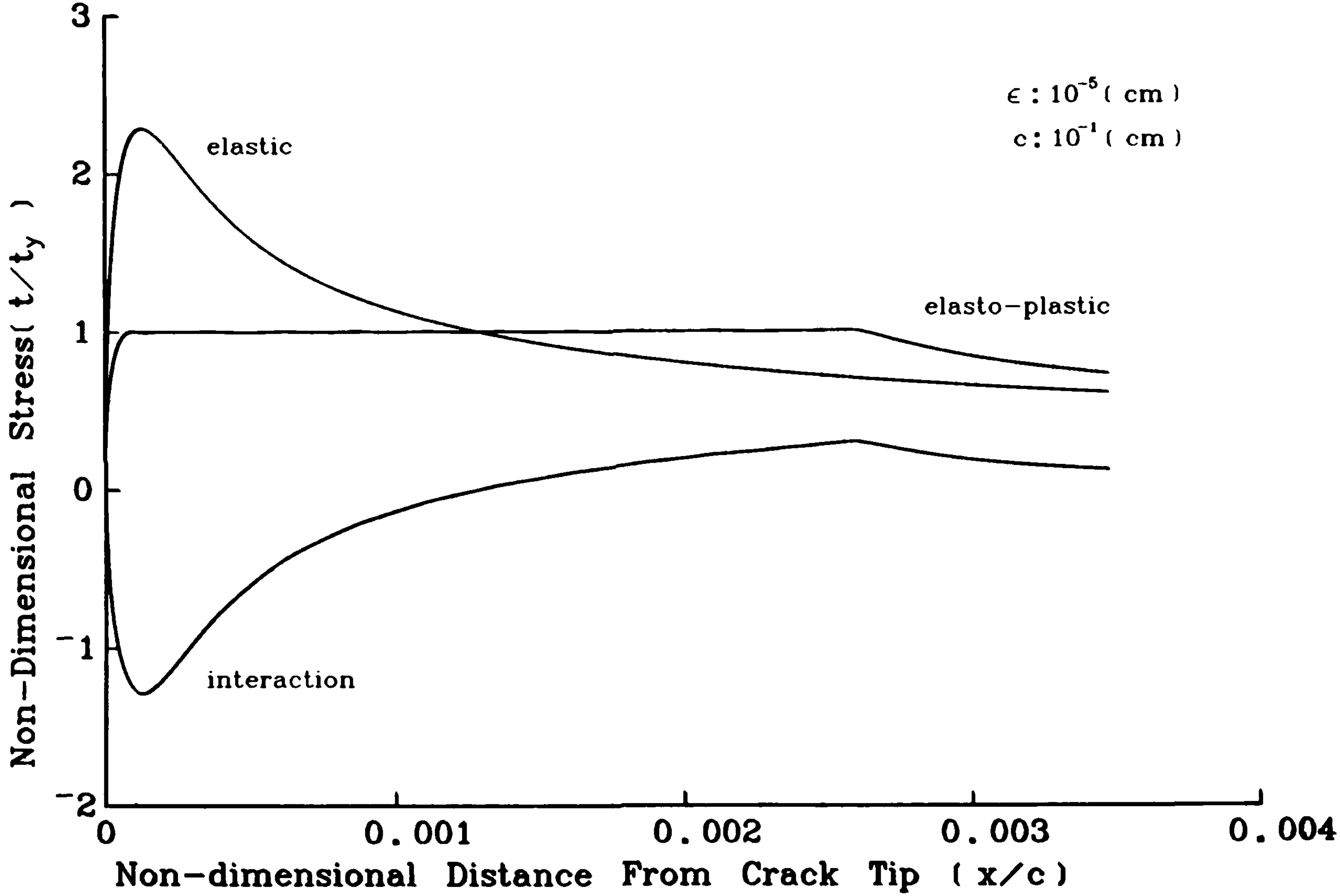

Fig. 12a. Stress Distribution Near Crack Tip (R = 0.051)

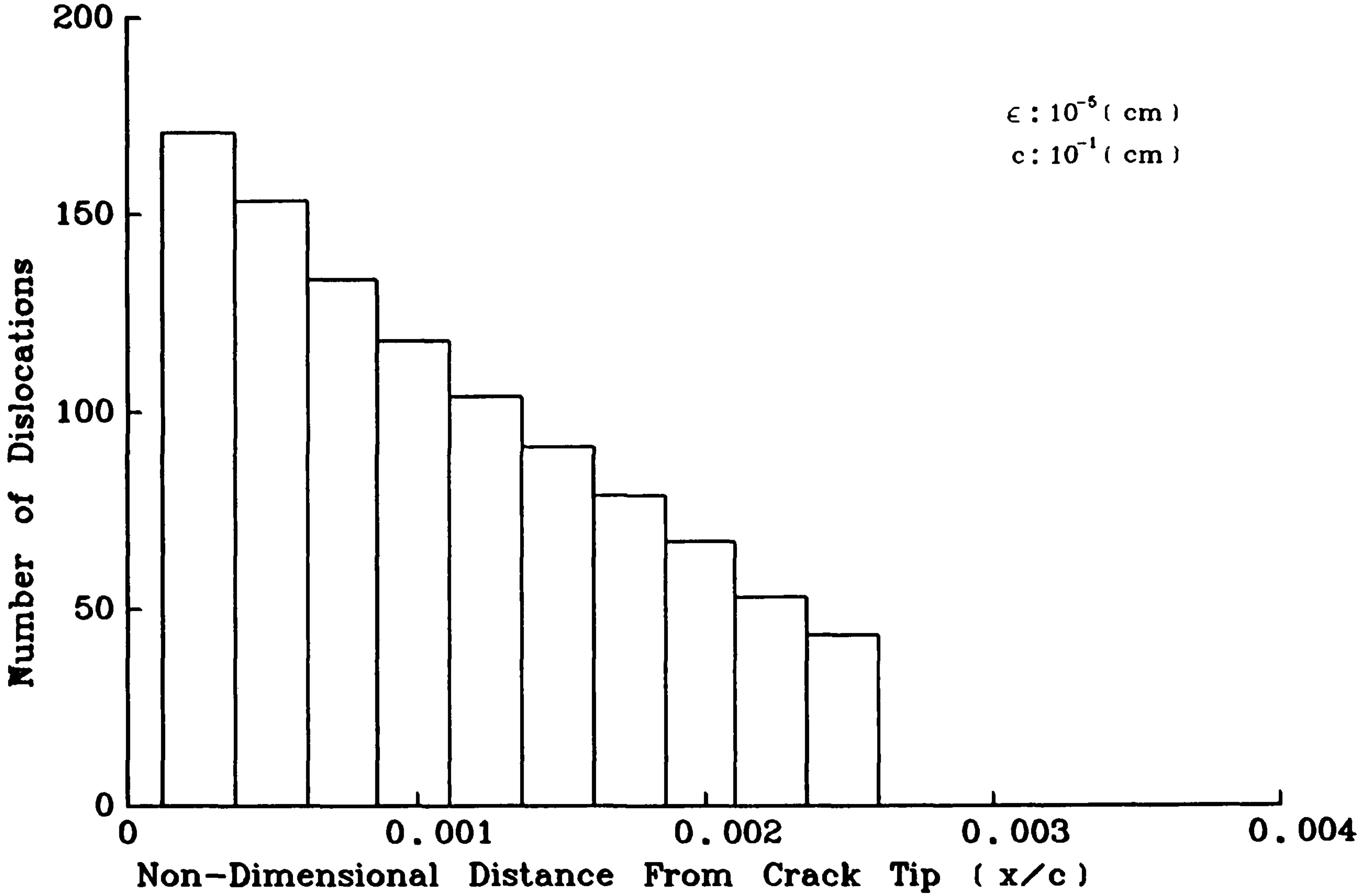

Fig. 12b. Dislocation Distribution Near Crack Tip (R = 0.051)

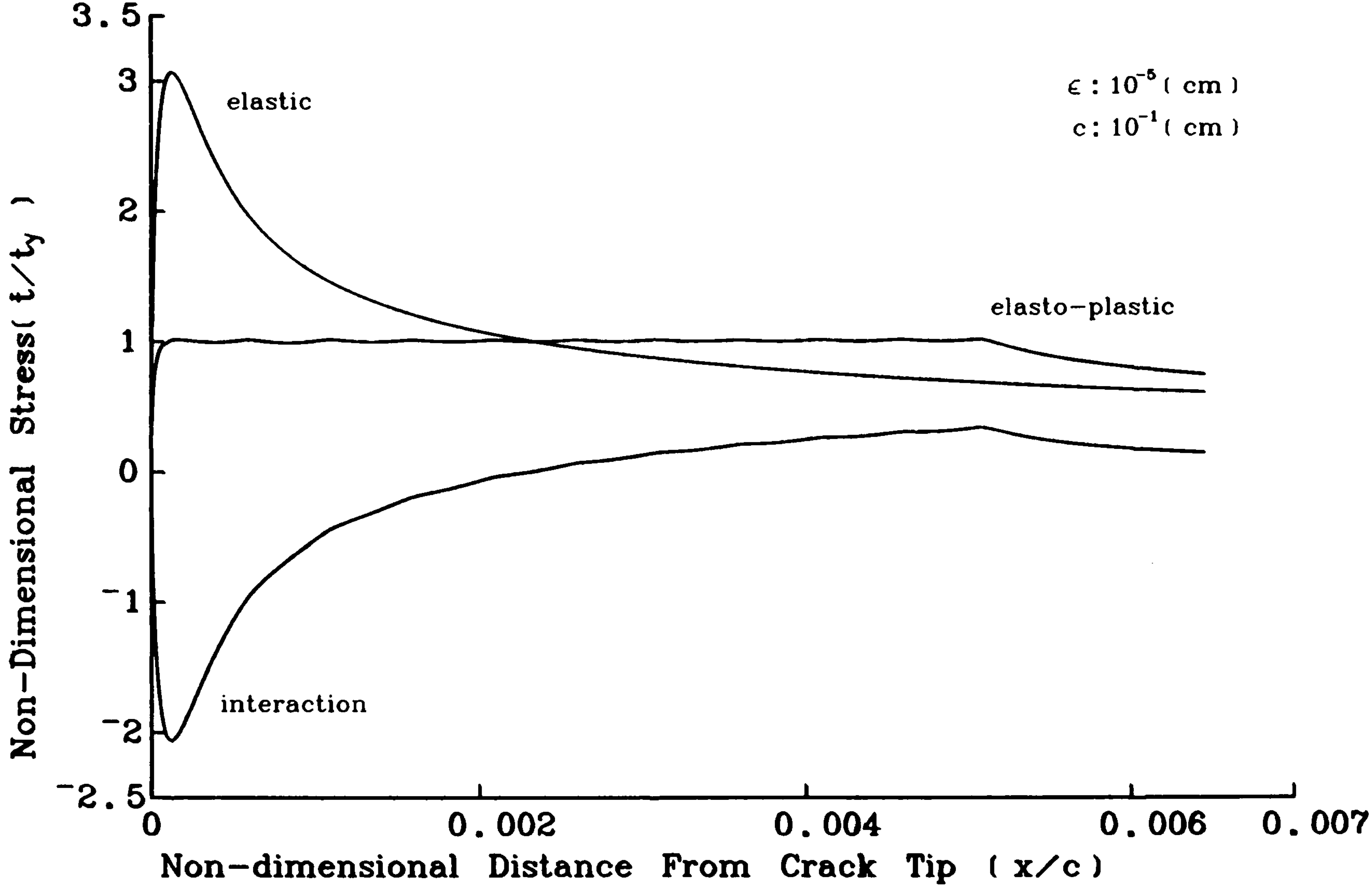

Fig. 13a. Stress Distribution Near Crack Tip (R = 0.068)

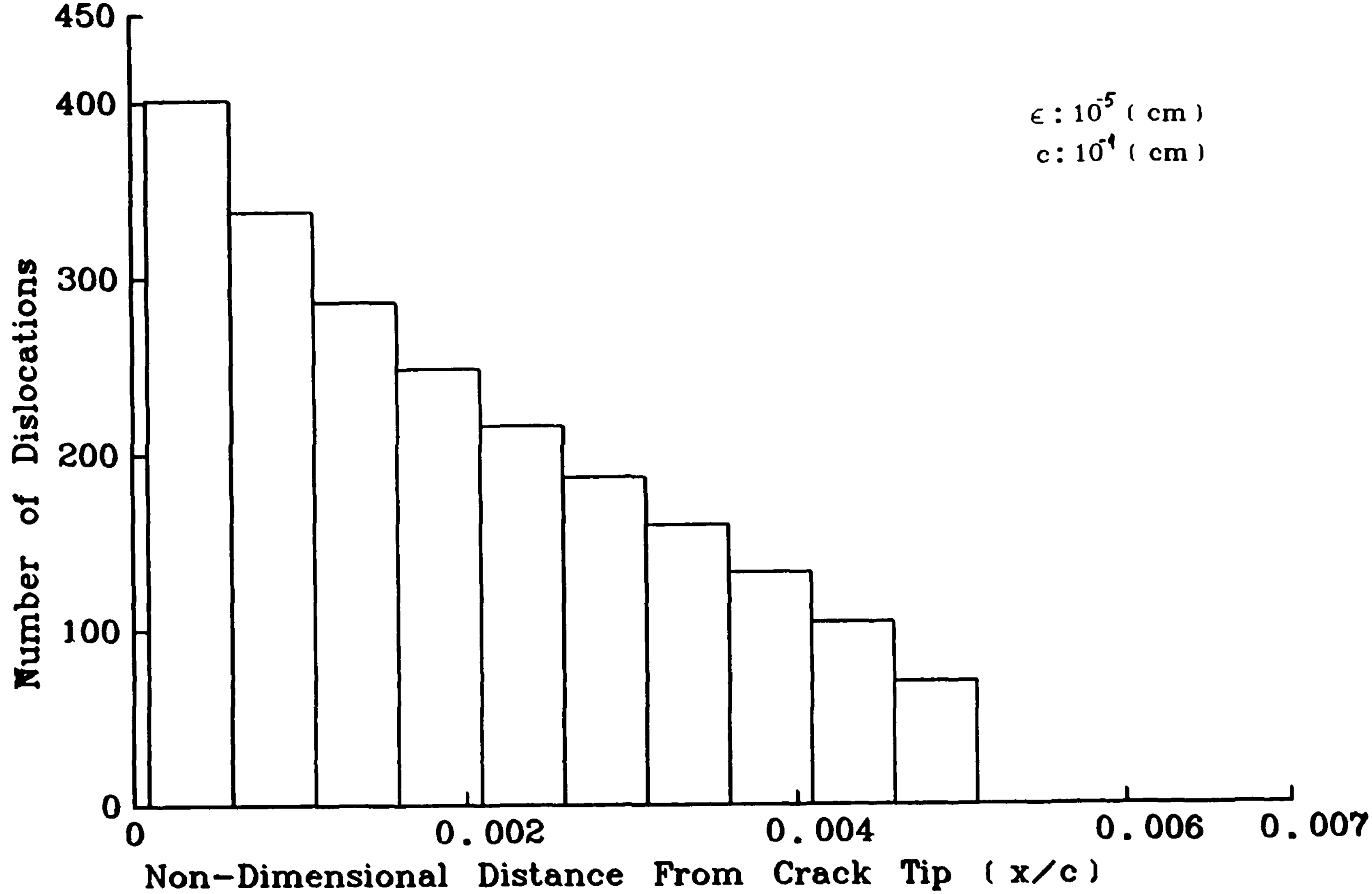

Fig. 13b. Dislocation Distribution Near Crack Tip (R = 0.068)

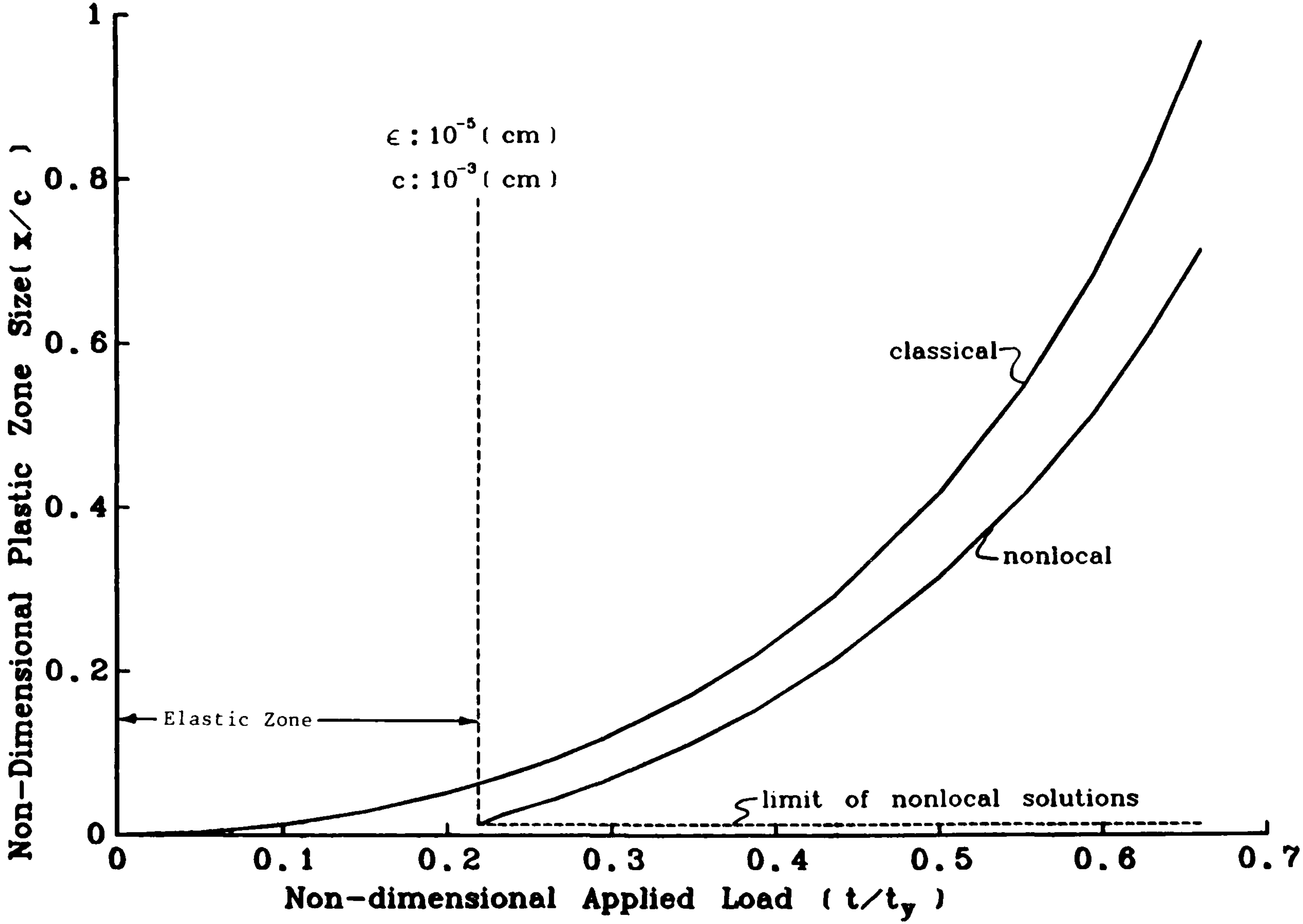

Fig. 14. The Variation of End Coordinates of Plastic Zones.

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